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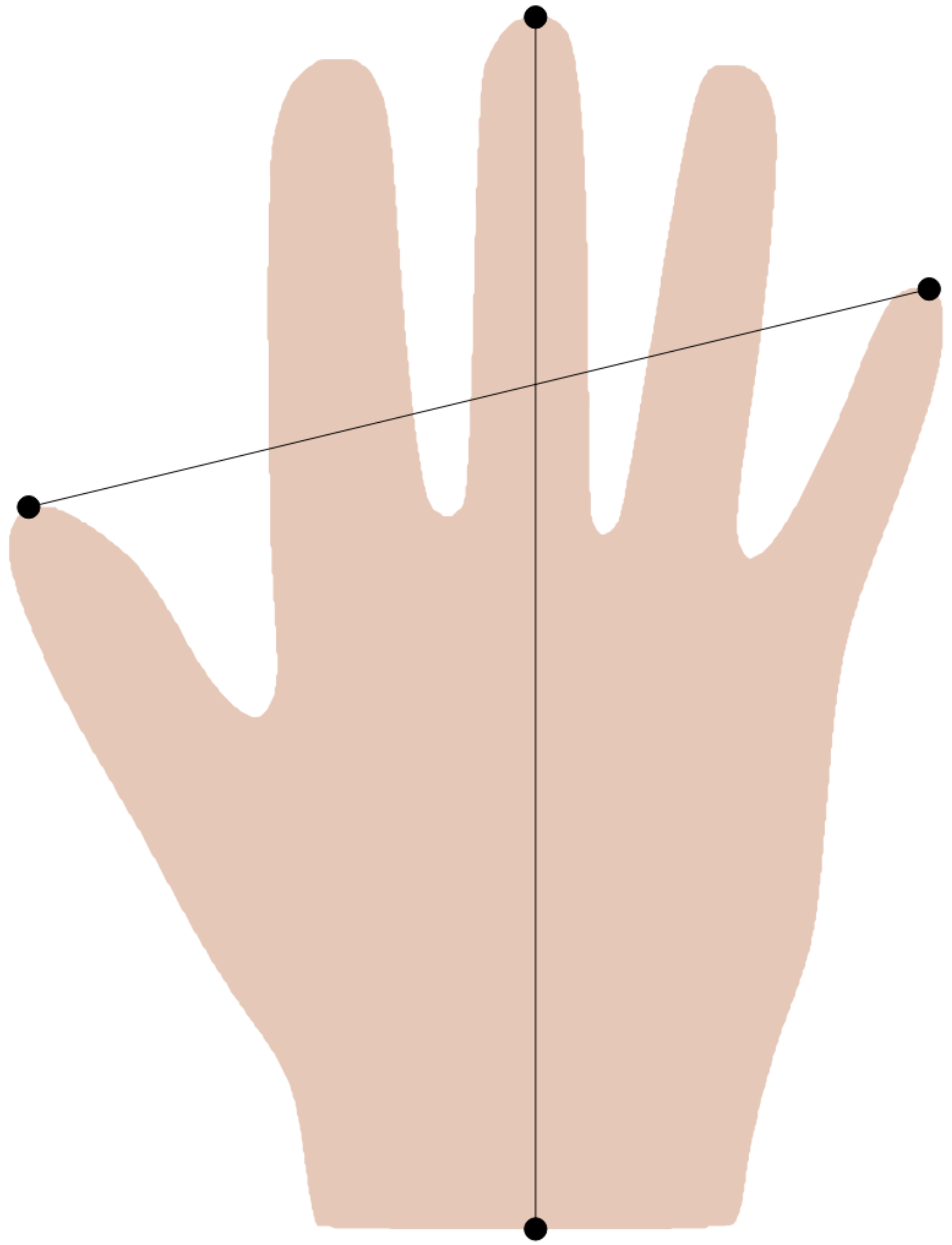
Modelling shape deformations

Measurements



Measurements

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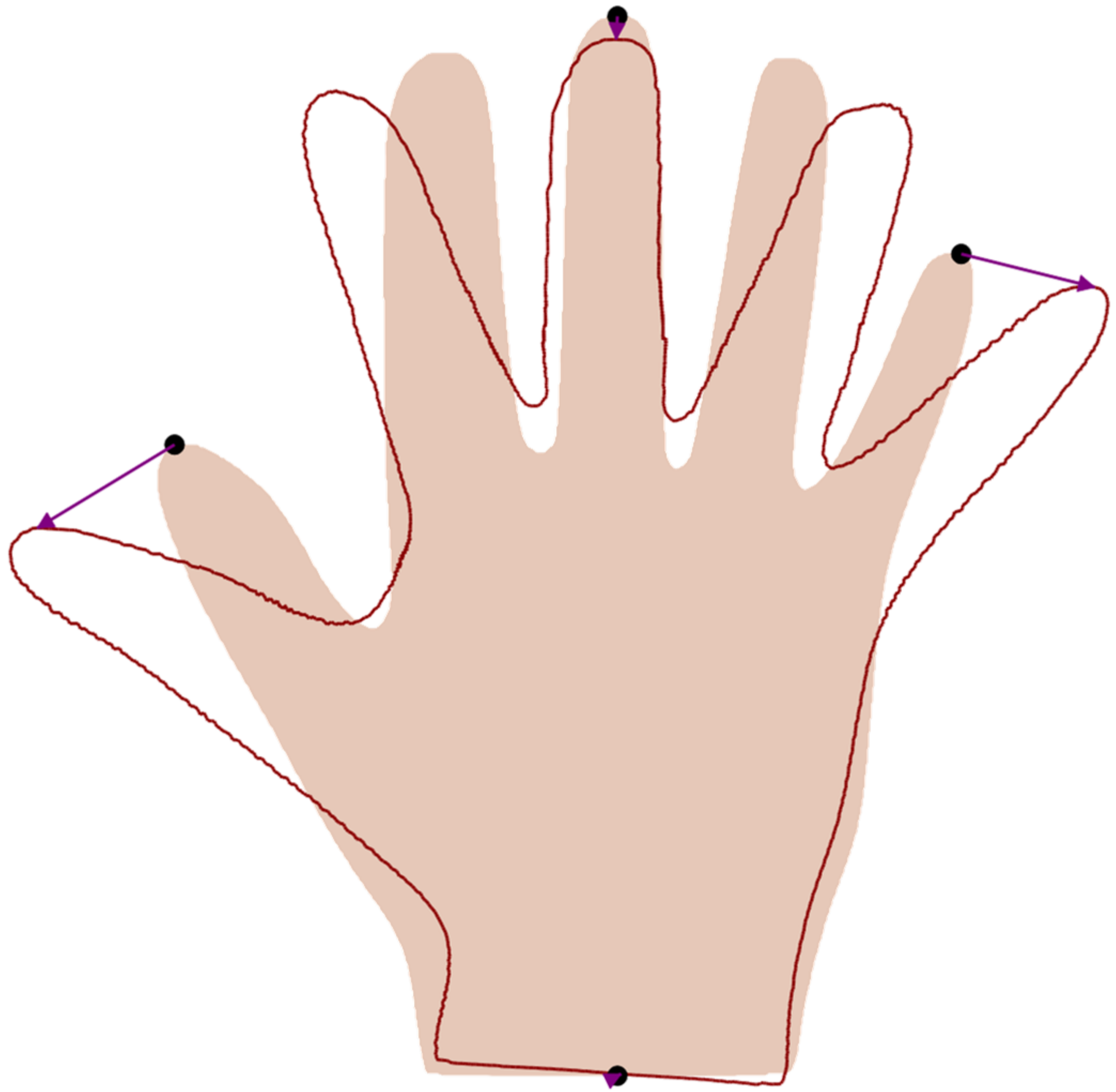

Measurements

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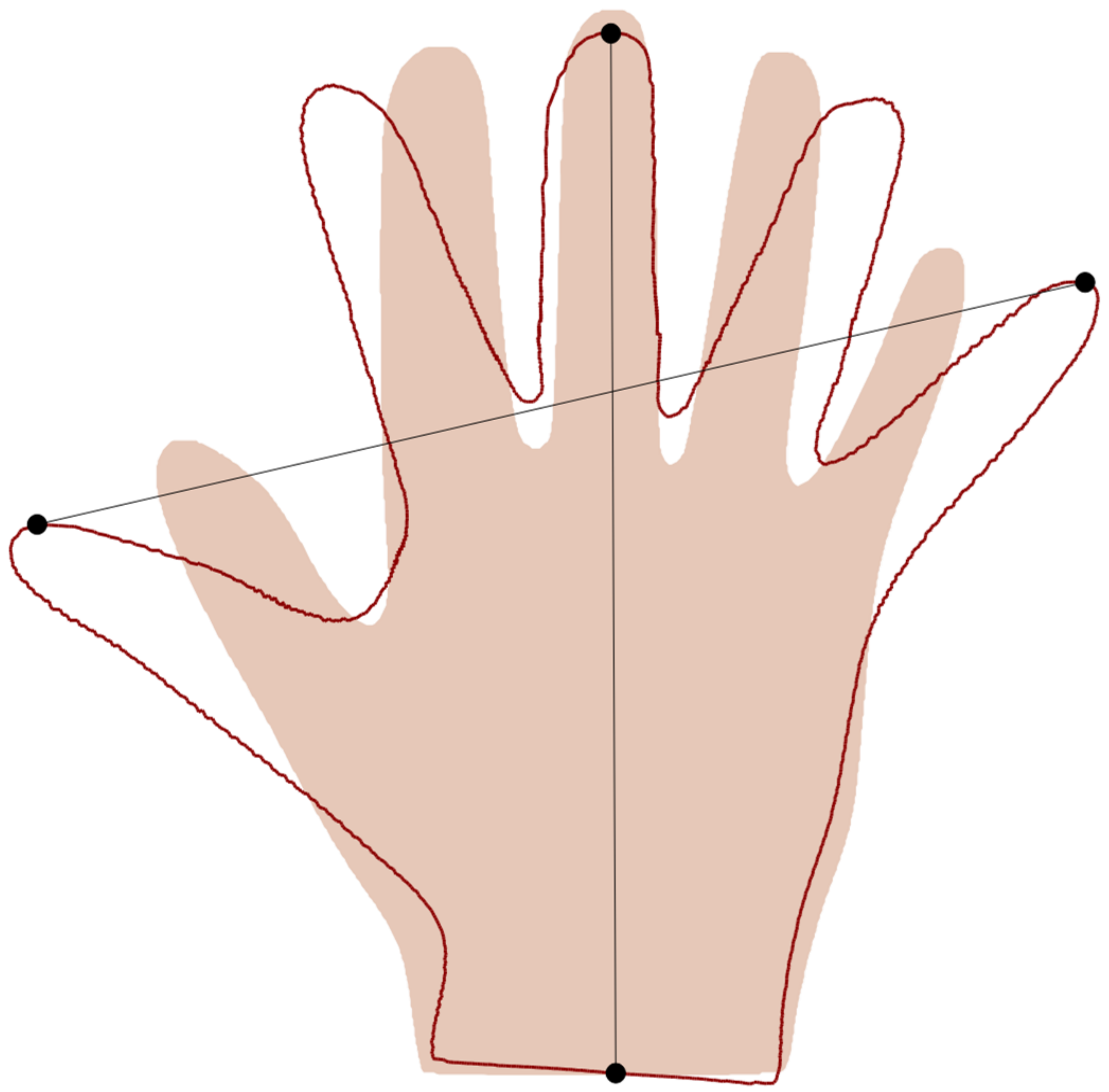

Shape changes as deformations

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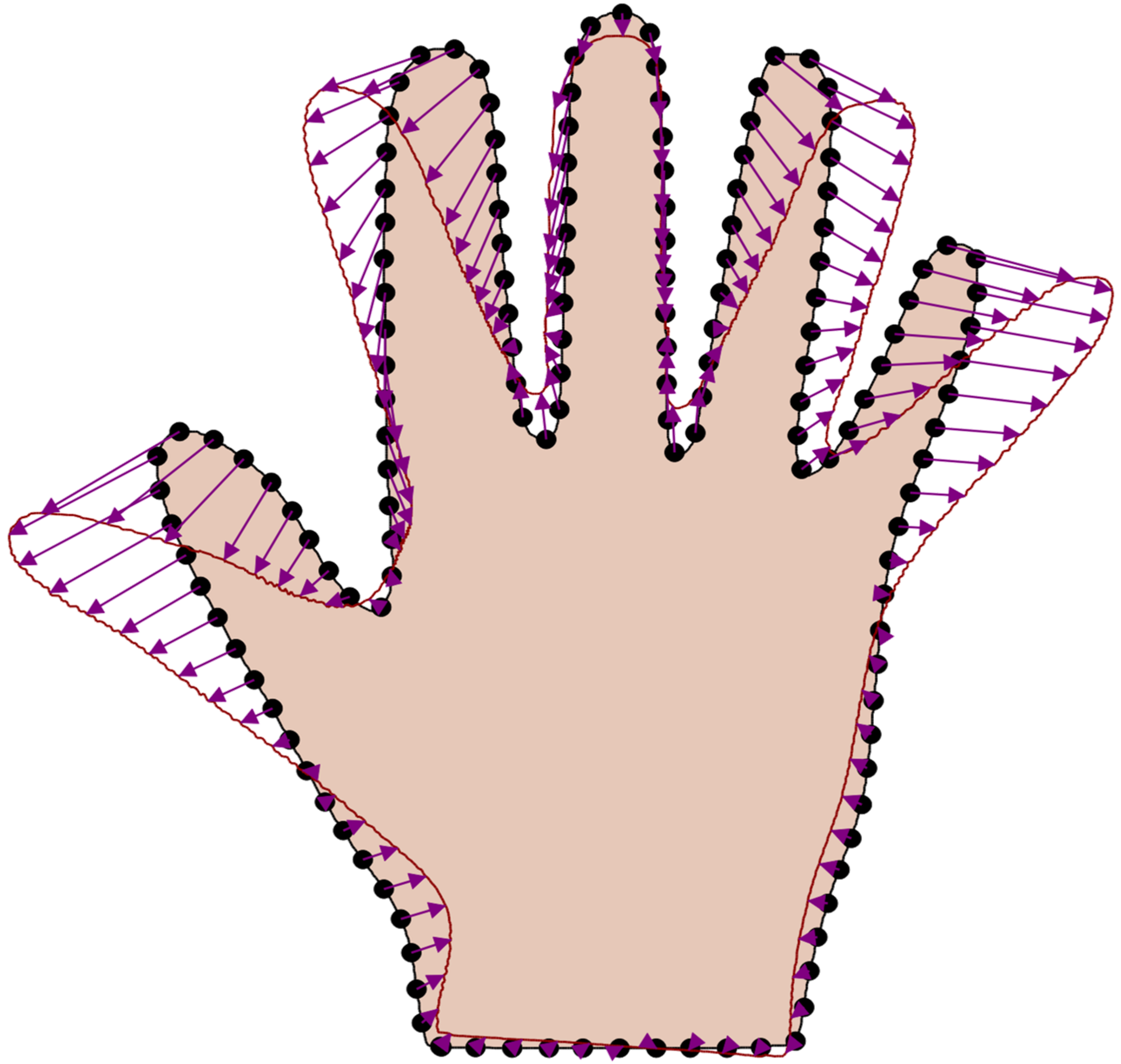

Shape changes as deformations

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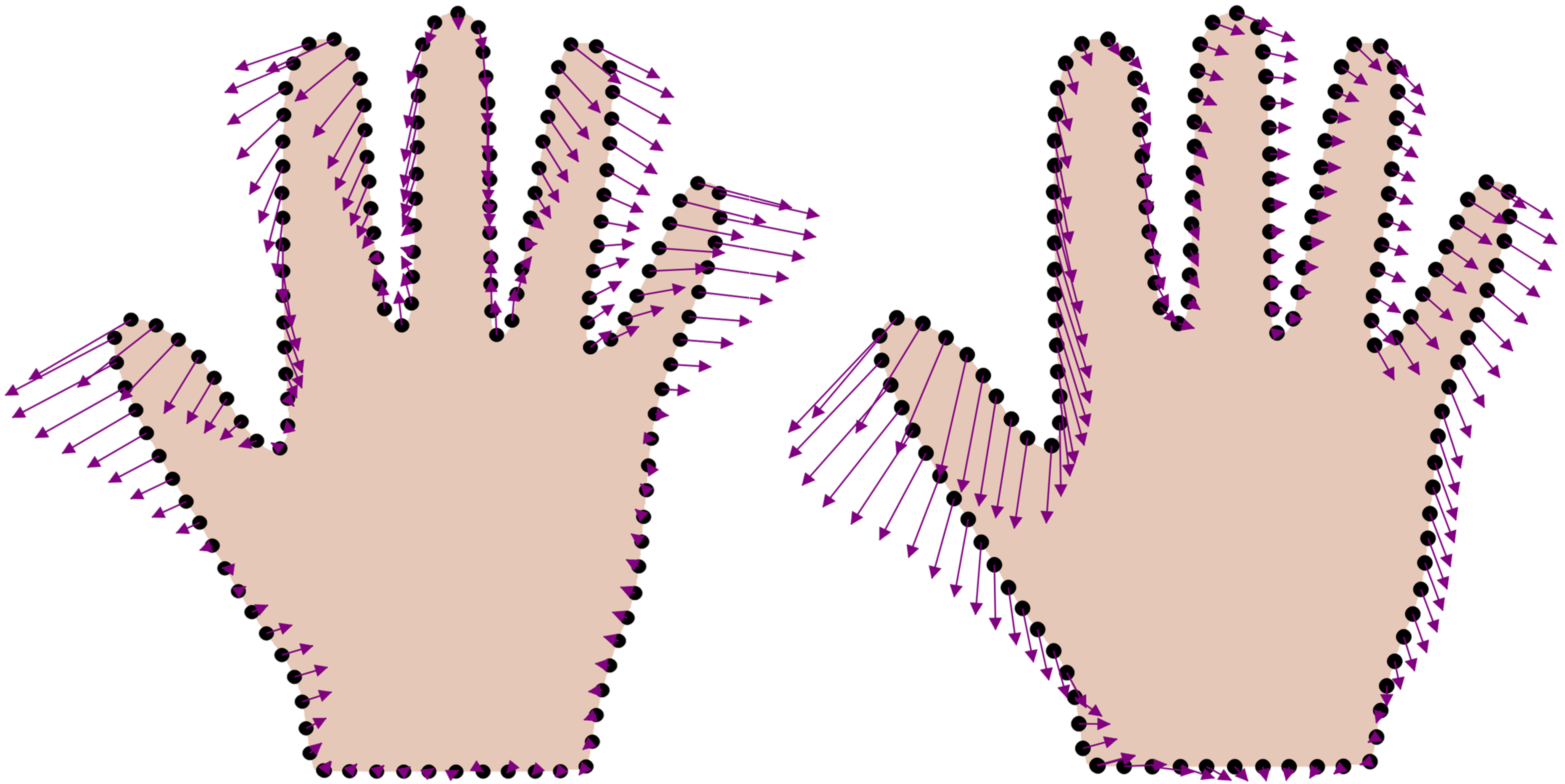

Shape changes as deformations

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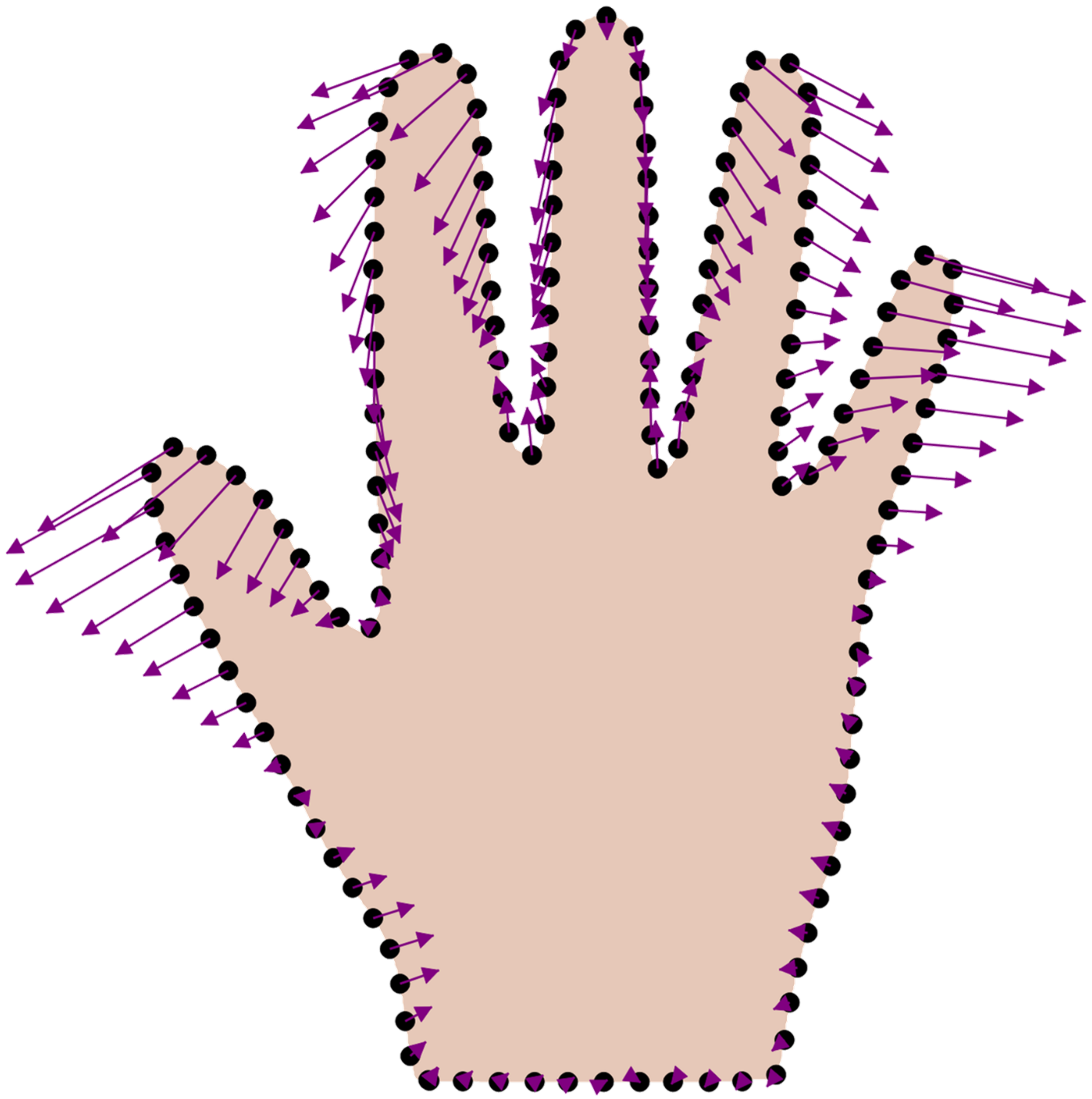
- Set of points defining a reference shape:

$$\Gamma_R = \{x \mid x \in \mathbb{R}^2\}$$

- Vector field modelling the deformations

$$u : \Gamma_R \rightarrow \mathbb{R}^2$$



Modelling possible deformations

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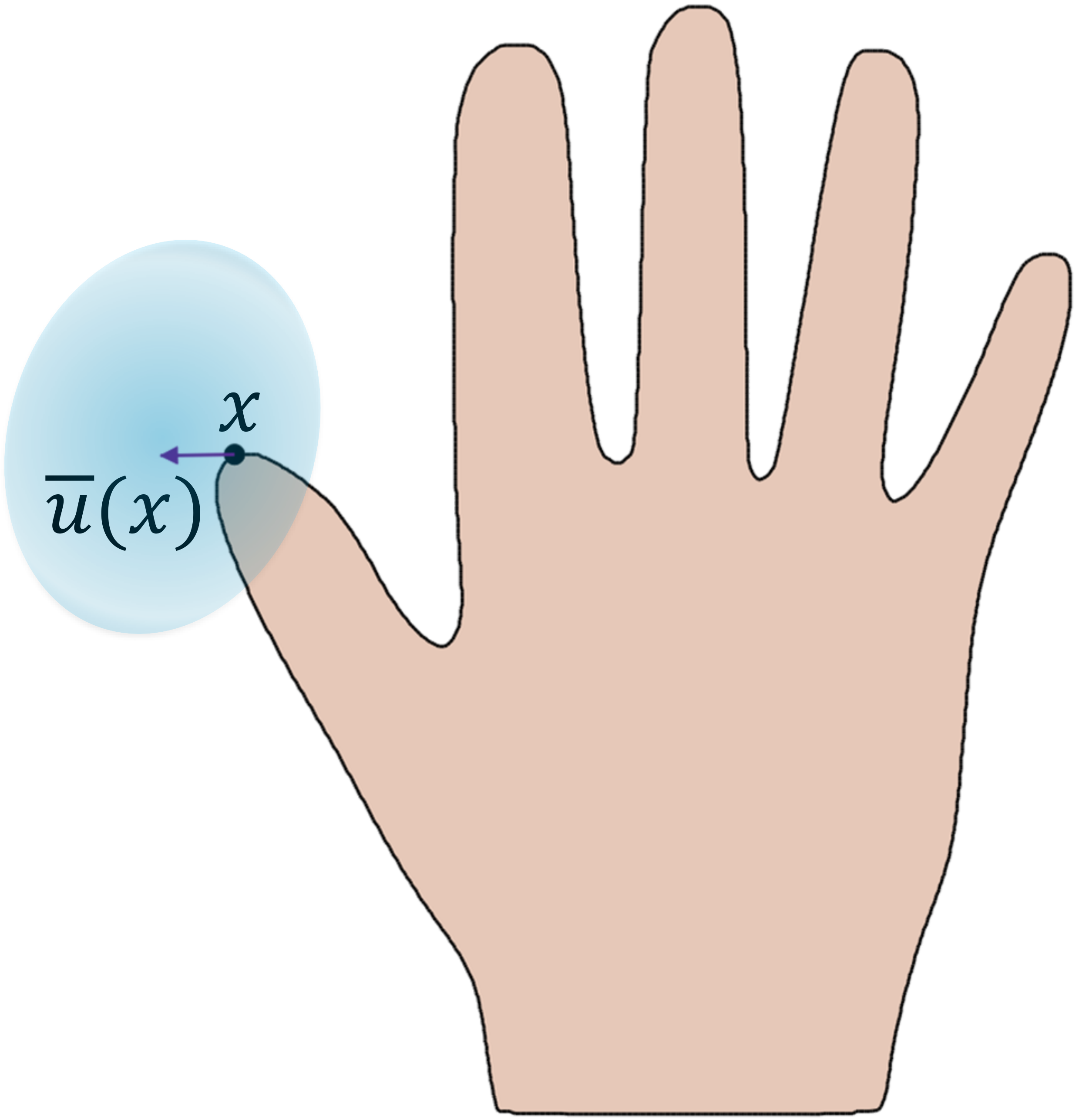
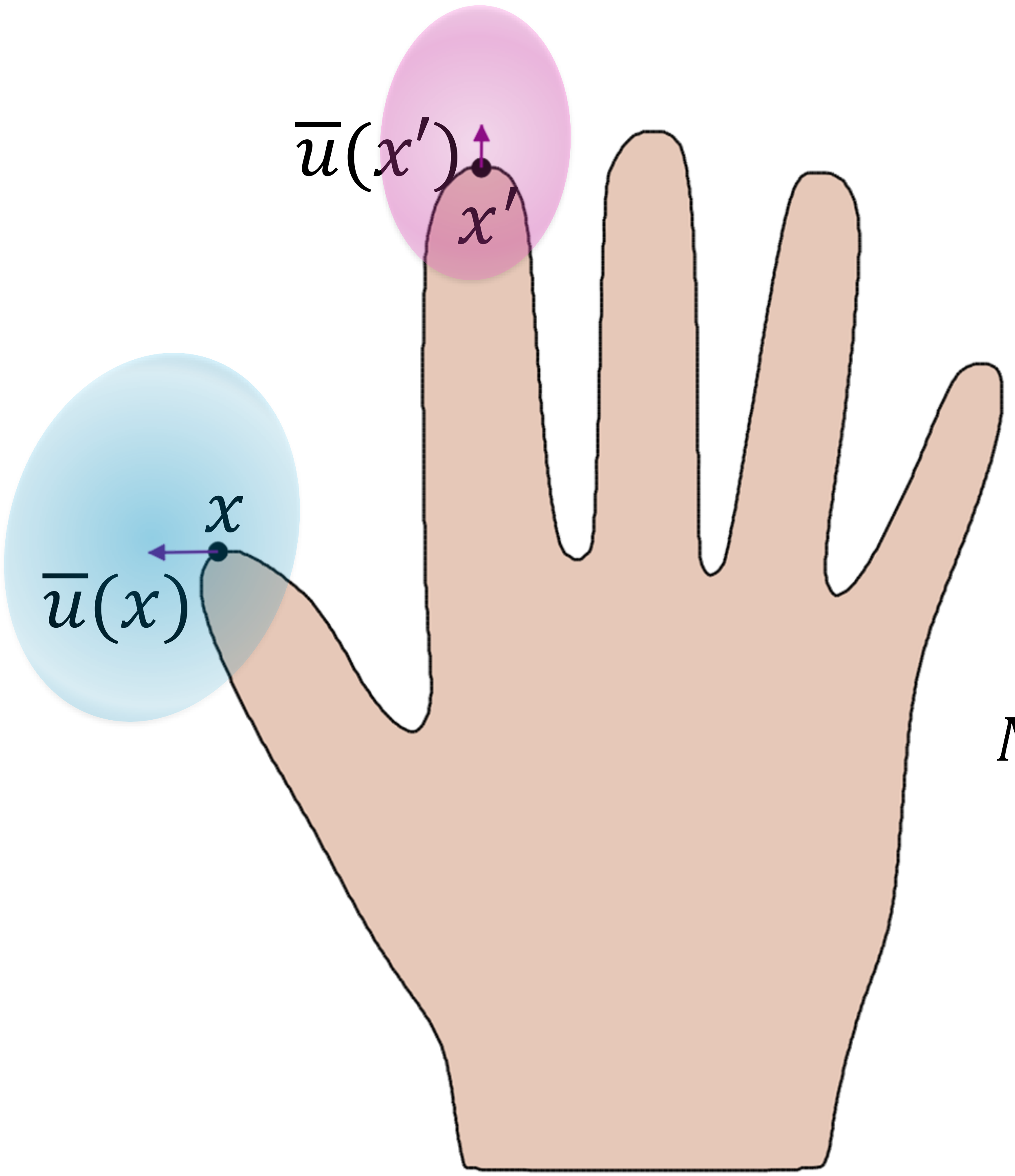


$$u(x) = \begin{pmatrix} u_1(x) \\ u_2(x) \end{pmatrix}$$

$$\sim N\left(\begin{pmatrix} \overline{u}_1(x) \\ \overline{u}_2(x) \end{pmatrix}, \begin{pmatrix} \Sigma_{11}(x) & \Sigma_{12}(x) \\ \Sigma_{21}(x) & \Sigma_{22}(x) \end{pmatrix} \right)$$

Modelling possible deformations



$$\begin{pmatrix} u(x) \\ u(x') \end{pmatrix} = \begin{pmatrix} u_1(x) \\ u_2(x) \\ u_1(x') \\ u_2(x') \end{pmatrix} \sim$$

$$N\left(\begin{pmatrix} \overline{u}_1(x) \\ \overline{u}_2(x) \\ \overline{u}_1(x') \\ \overline{u}_2(x') \end{pmatrix}, \begin{pmatrix} \Sigma_{11}(x,x) & \Sigma_{12}(x,x) & \Sigma_{11}(x,x') & \Sigma_{12}(x,x') \\ \Sigma_{21}(x,x) & \Sigma_{22}(x,x) & \Sigma_{21}(x,x') & \Sigma_{22}(x,x') \\ \Sigma_{11}(x',x) & \Sigma_{12}(x',x) & \Sigma_{11}(x',x') & \Sigma_{12}(x',x') \\ \Sigma_{21}(x',x) & \Sigma_{22}(x',x) & \Sigma_{21}(x',x') & \Sigma_{22}(x',x') \end{pmatrix}\right)$$

Modelling possible deformations

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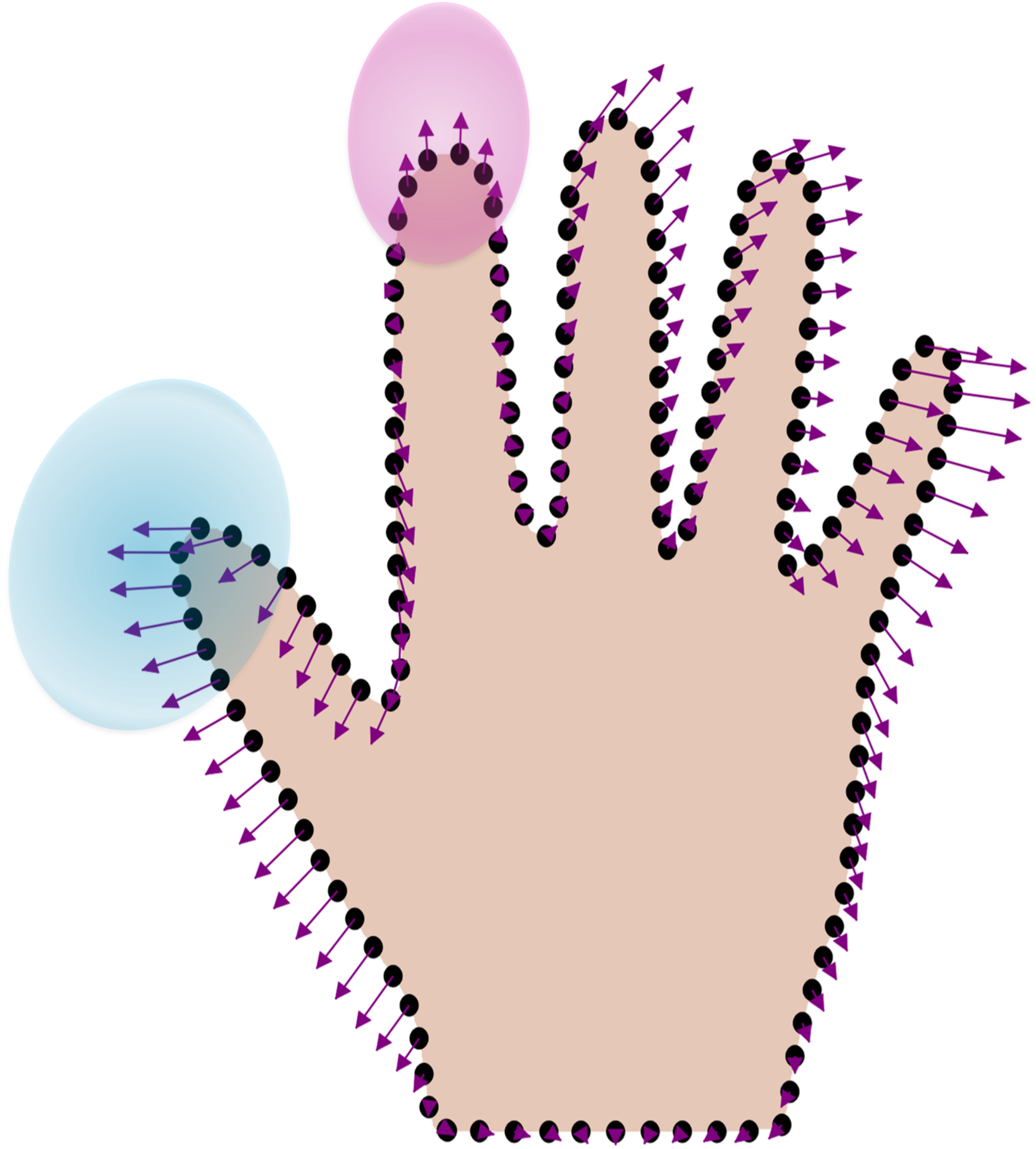

Assume we have

- Mean function: $\mu: \Gamma_R \to \mathbb{R}^2$
- Covariance function: $k: \Gamma_R \times \Gamma_R \to \mathbb{R}^{2\times 2}$

For any finite set Γ_R we can define
$u \sim N(\vec{\mu}, K)$, with

$$\vec{\mu} = \big(\mu(x)\big)_{x\in\Gamma_R}$$

$$K = \big(k(x,x')\big)_{x,x'\in\Gamma_R}$$

Modelling possible deformations

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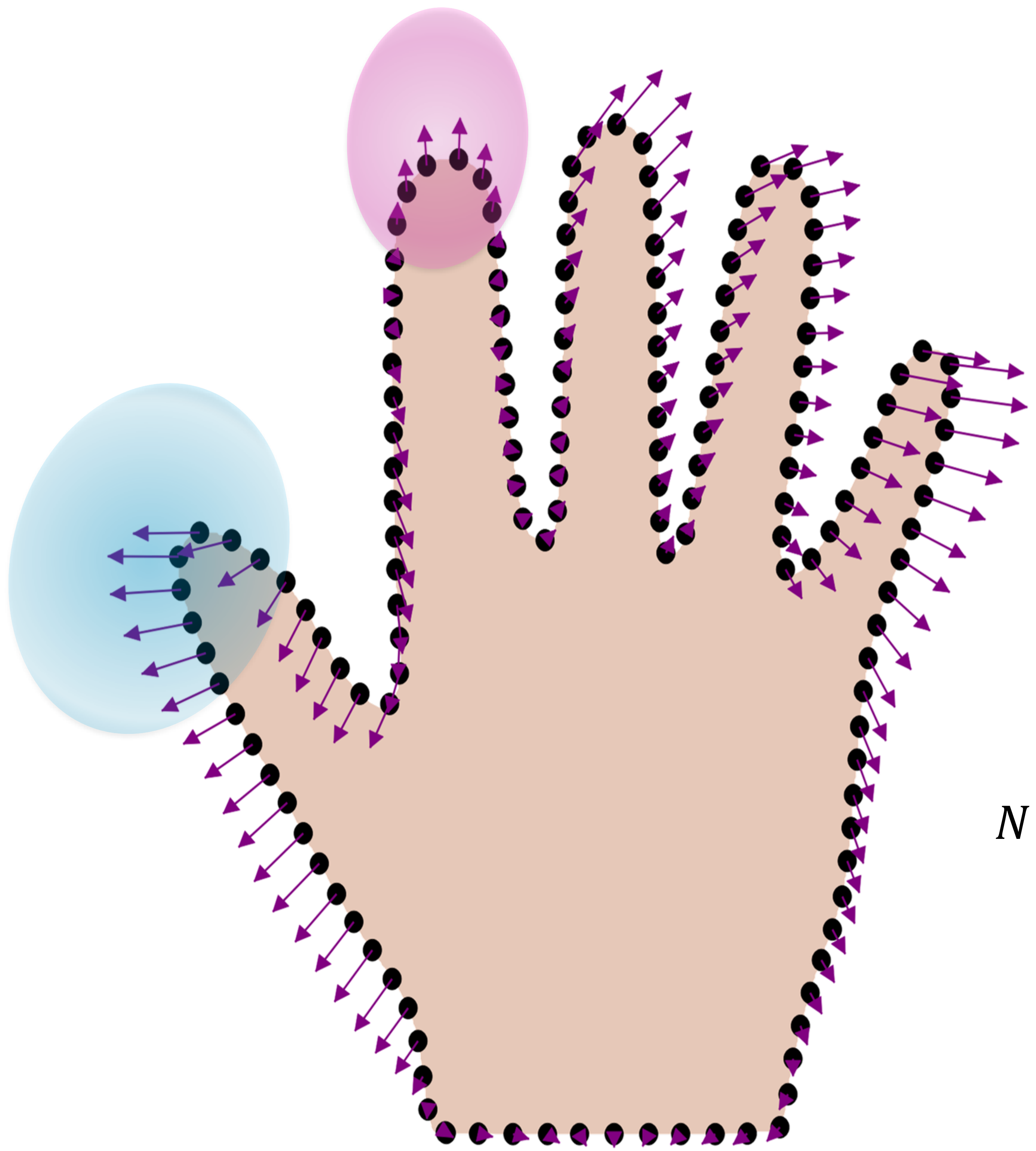

$$
\begin{pmatrix} \vdots \\ u(x) \\ \vdots \\ u(x') \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ u_1(x) \\ u_2(x) \\ \vdots \\ u_1(x') \\ u_2(x') \\ \vdots \end{pmatrix} \sim
$$

$$
N\left(\begin{pmatrix} \vdots \\ \mu_1(x) \\ \mu_2(x) \\ \vdots \\ \mu_1(x') \\ \mu_2(x') \\ \vdots \end{pmatrix}, \begin{pmatrix} & \vdots & & & \vdots & \\ \cdots & k_{11}(x,x) & k_{12}(x,x) & \cdots & k_{11}(x,x') & k_{12}(x,x') & \cdots \\ & k_{21}(x,x) & k_{22}(x,x) & & k_{21}(x,x') & k_{22}(x,x') & \\ & \vdots & & \ddots & \vdots & \\ \cdots & k_{11}(x',x) & k_{12}(x',x) & & k_{11}(x',x') & k_{12}(x',x') & \cdots \\ & k_{21}(x',x) & k_{22}(x',x) & \cdots & k_{21}(x',x') & k_{22}(x',x') & \\ & \vdots & & & \vdots & \end{pmatrix} \right)
$$

Gaussian processes

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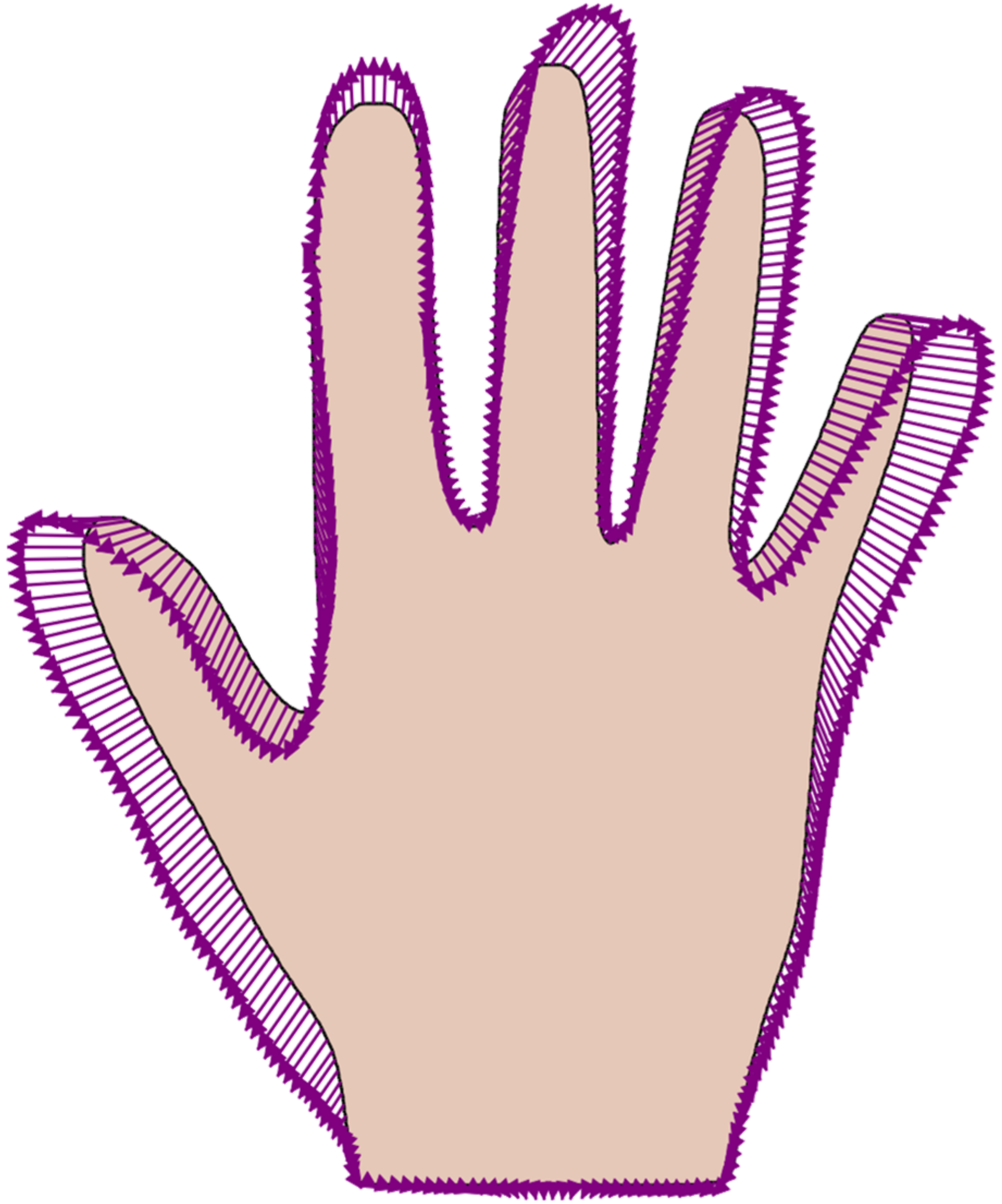

We can define the Gaussian process

$$u \sim GP(\mu, k)$$

with mean function

$$\mu: \Gamma_R \to \mathbb{R}^2$$

and covariance function

$$k: \Gamma_R \times \Gamma_R \to \mathbb{R}^{2\times 2}$$

on arbitrary (even infinite) sets Γ_R.

Gaussian processes

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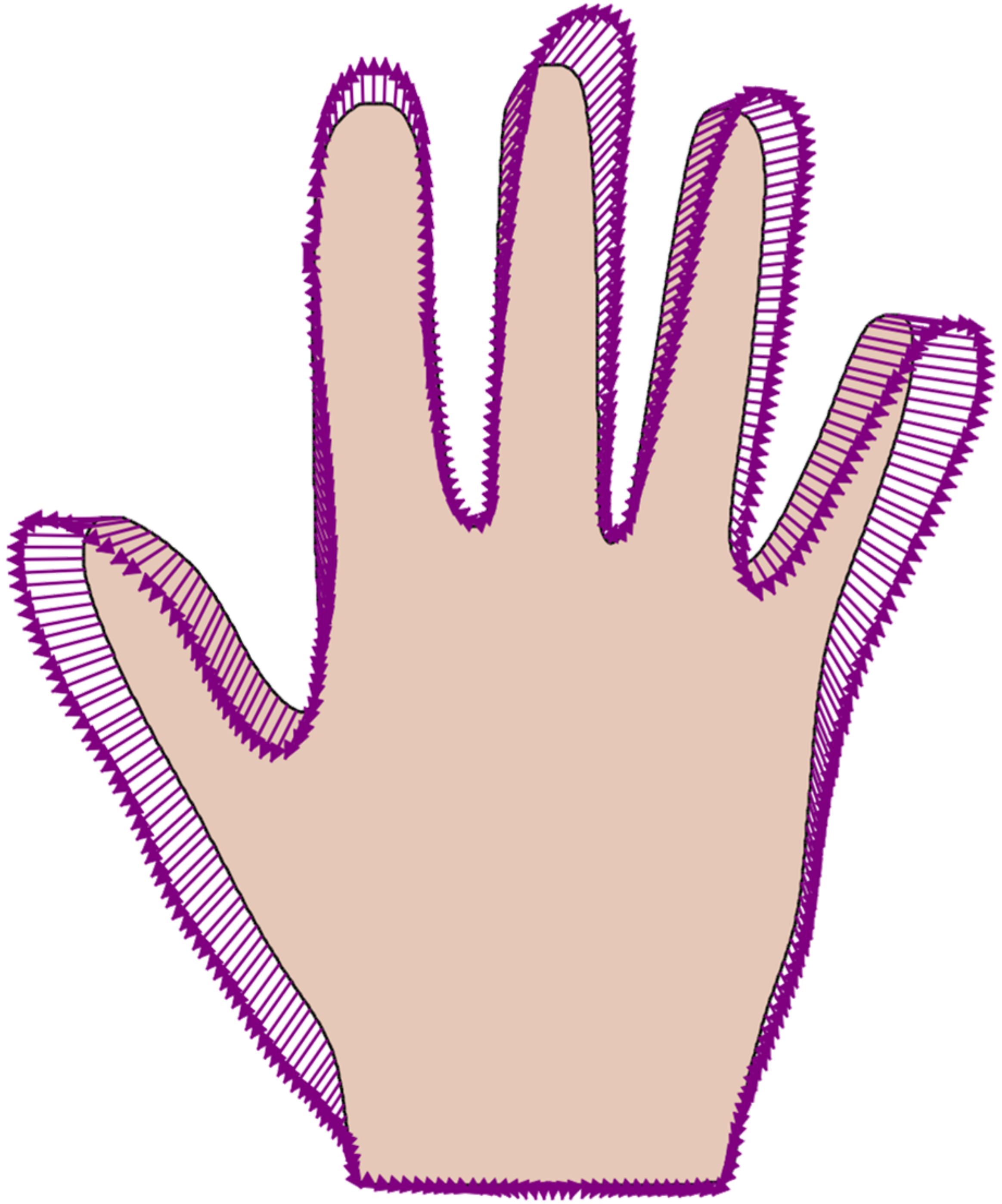

- Gaussian processes are
  - extensions of the multivariate normal distribution to distributions over functions
  - Well established in machine learning and statistics