University of Basel

Shape model fitting – A case study

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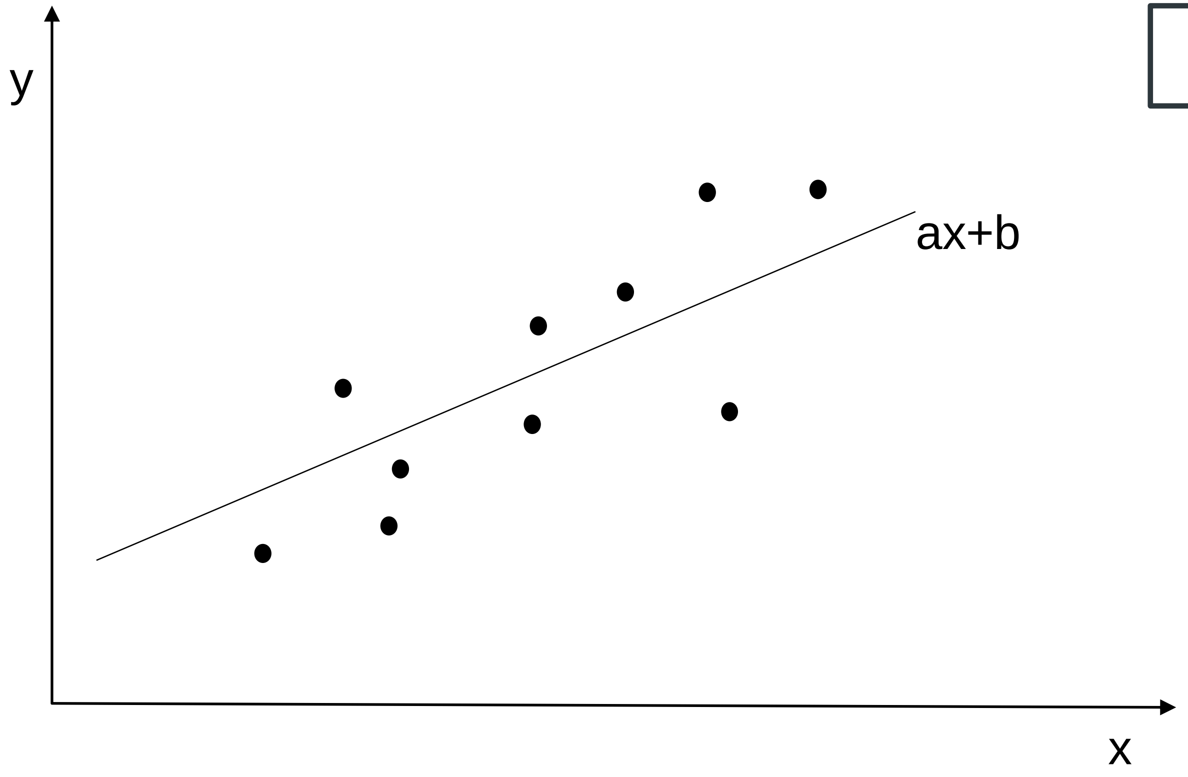
Bayesian linear regression - revisited



Likelihood

Priors

Model

$y(x) = ax + b + \epsilon$

$p(\, y_i | x_i, a, b, \sigma^2) = N(ax_i + b, \sigma^2)$

$a \sim N(1, 0.1)$

$b \sim N(0, 2)$

$\sigma^2 \sim logNormal(0, 0.25)$

Shape model fitting as Bayesian linear regression

Model

$u \sim GP(\mu, k) \;\Leftrightarrow\; u = \mu + \sum_i^r \alpha_i \sqrt{\lambda_i}\phi_i, \alpha_i \sim N(0,1)$

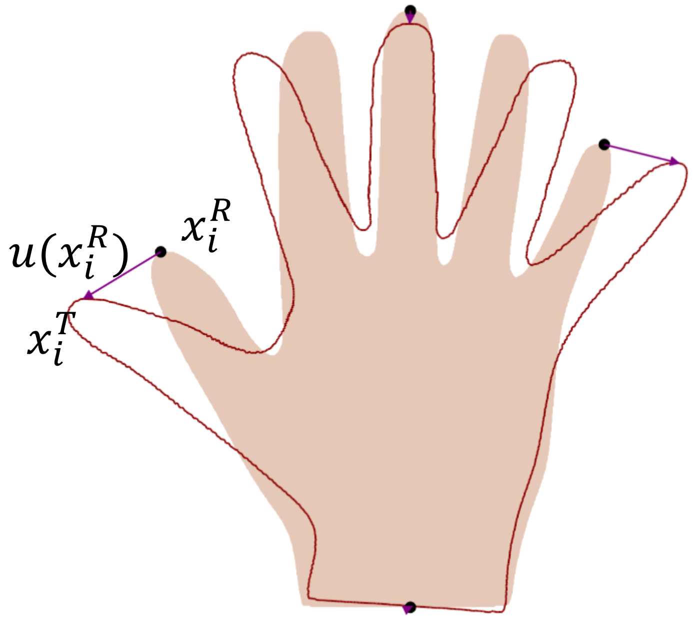
Likelihood

$p\left(x_i^T \middle| x_i^R, \alpha_1 \dots \alpha_n, \sigma^2\right) = N\left(x_i^R + u[\alpha](x_i^R), \sigma^2 I_{3\times 3}\right)$

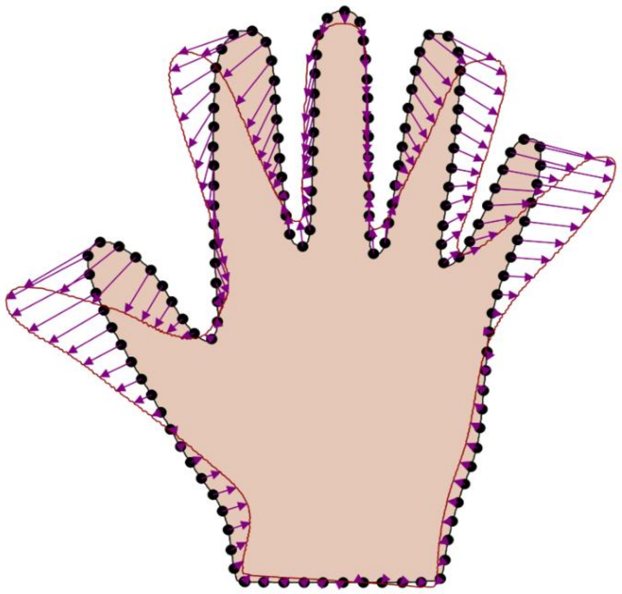
Priors

$\alpha_i \sim N(0,1), i = 1, \dots, r$

$\sigma^2 \sim logNormal(0, 0.25)$



$$u[\alpha](x) \; = \mu(x) + \sum_i^r \alpha_i \sqrt{\lambda_i}\phi_i(x)$$



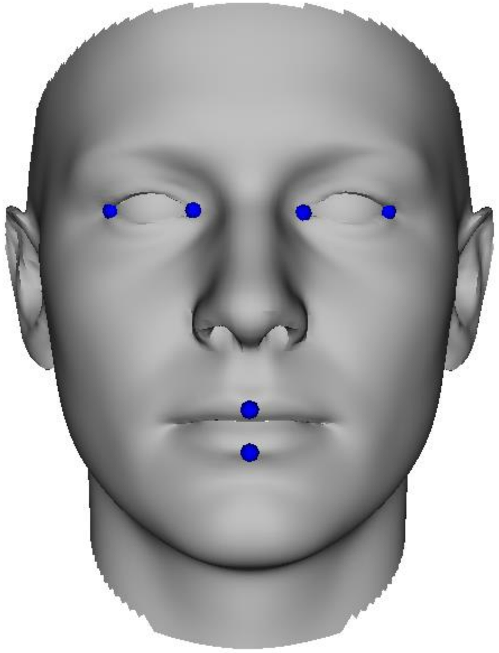
Case study

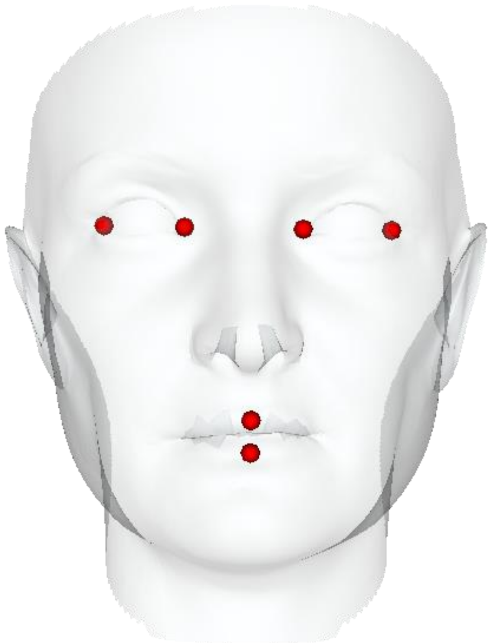
Model

$u \sim GP(\mu, k) \iff u = \mu + \sum_i^r \alpha_i \sqrt{\lambda_i} \phi_i, \alpha_i \sim N(0,1)$

$p\left(x_i^T \middle| x_i^R, \alpha_1 \dots \alpha_n, \sigma^2\right) = N\left(x_i^R + u[\alpha](x_i^R), \sigma^2 I_{3\times 3}\right)$

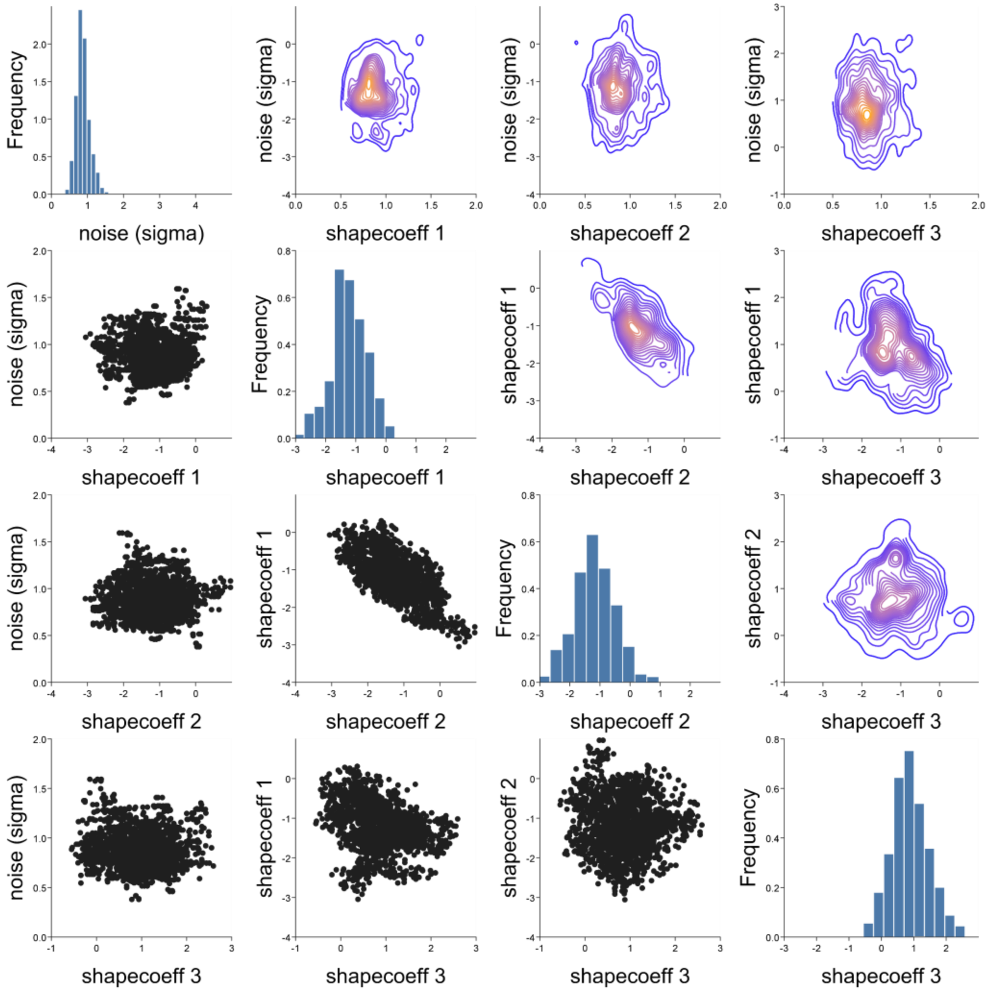
$\alpha_i \sim N(0,1), i = 1, \dots, r$
$\sigma^2 \sim logNormal(0, 0.25)$



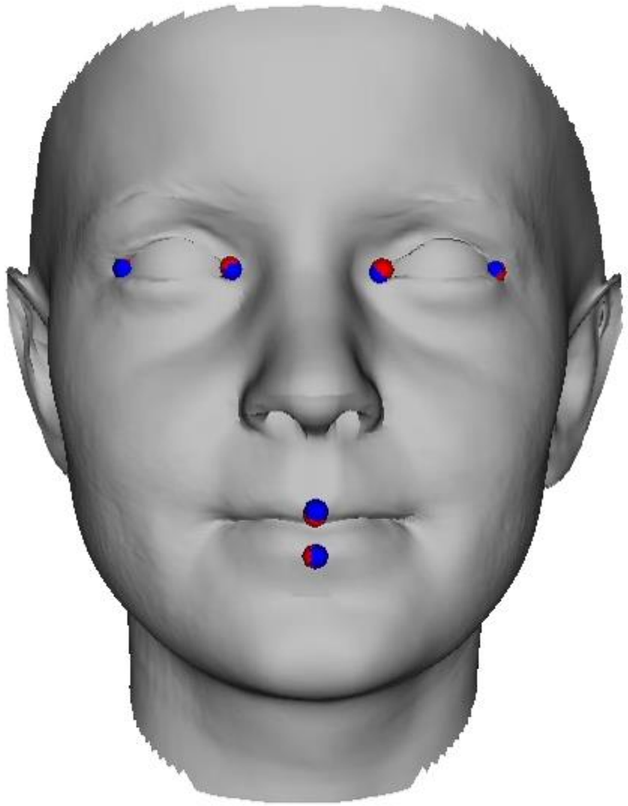


Posterior distribution

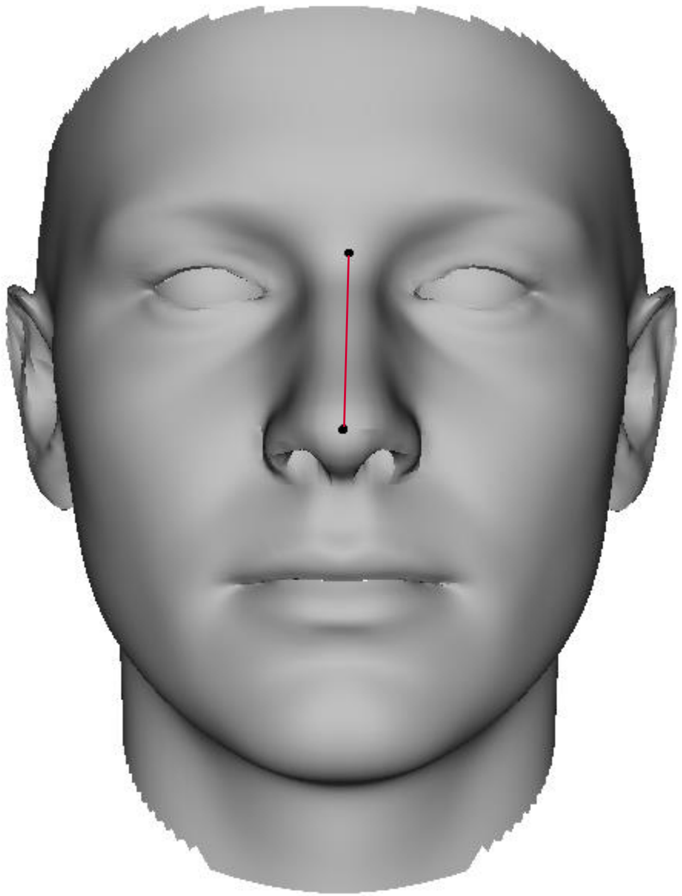
$$p(\alpha_1, \ldots, \alpha_r, \sigma^2 | x_1^R, \ldots, x_n^R, x_1^T, \ldots, x_n^T)$$

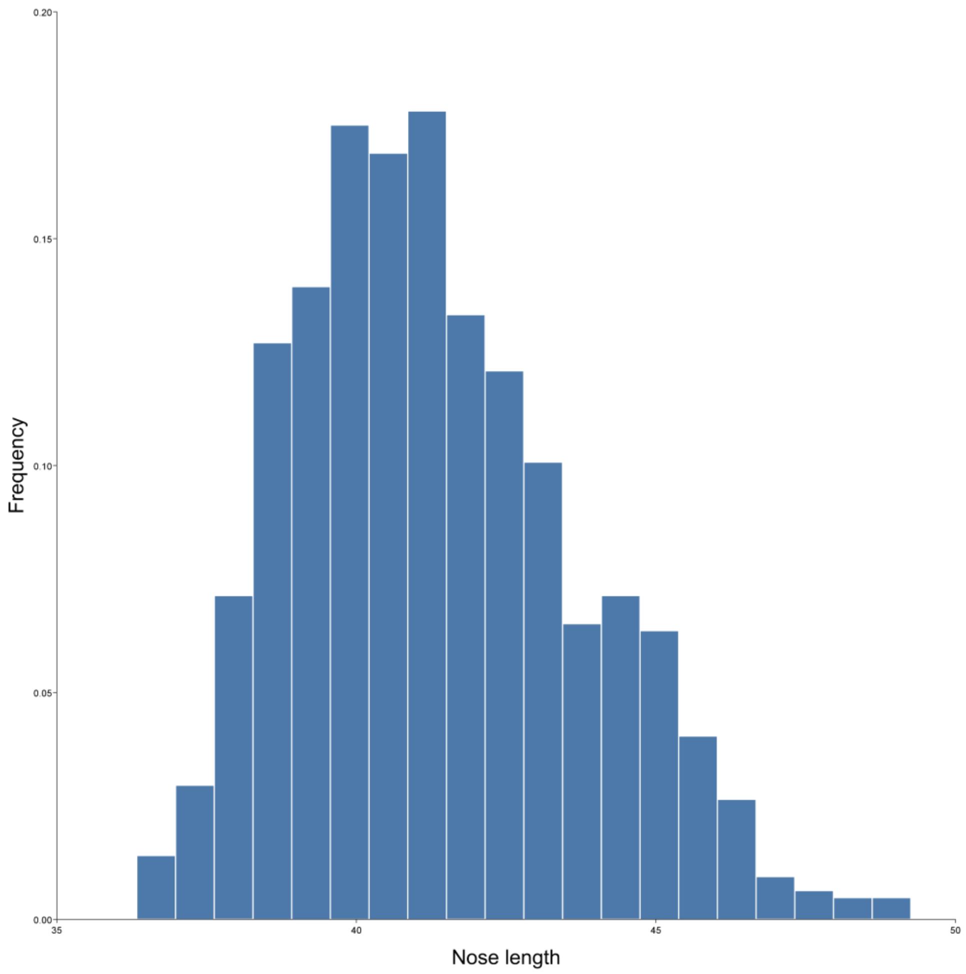


Posterior shapes

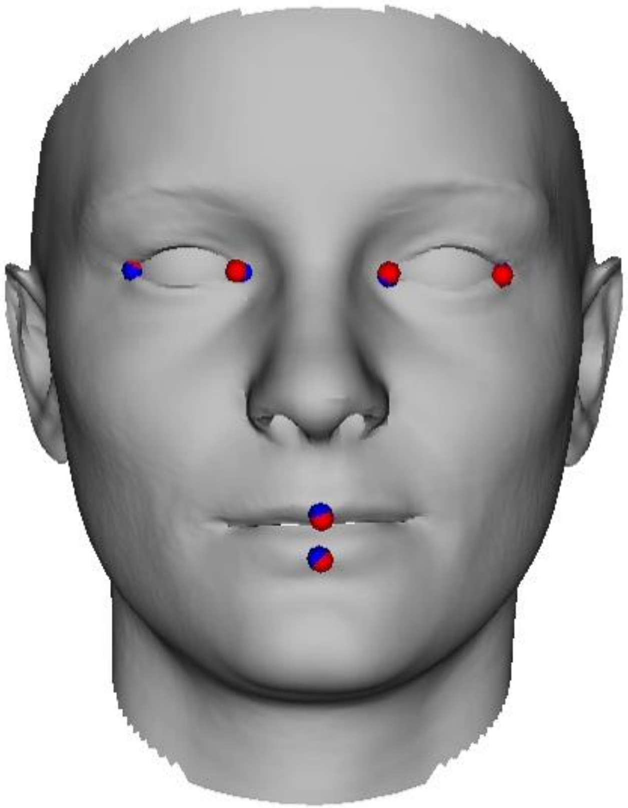


Measurements on posterior shape





Including pose



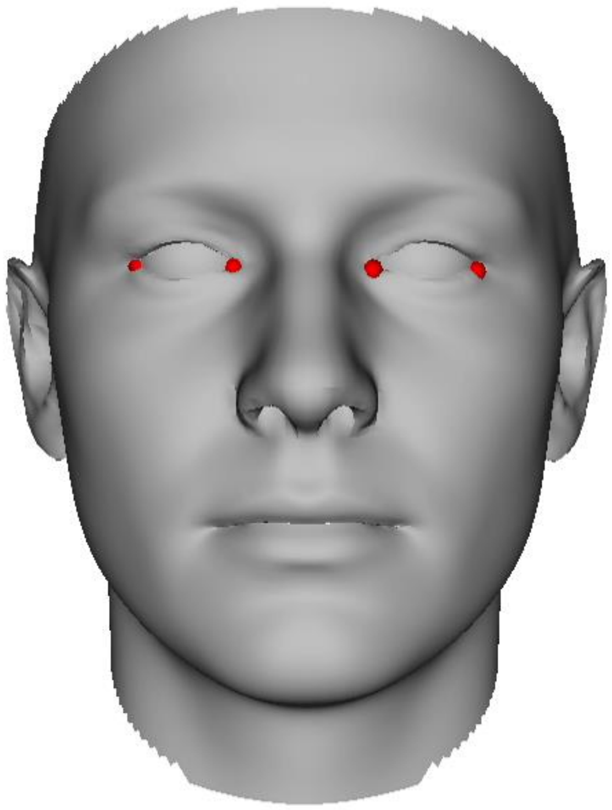
Model

$$p\left(x_i^T \,\middle|\, x_i^R, \alpha_1 \ldots \alpha_n, \sigma^2, \phi, \theta, \psi, t_x, t_y, t_z\right) = N\left(R_{\phi,\theta,\psi}\left[x_i^R + u[\alpha]\left(x_i^R\right)\right] + t, \sigma^2 I_{3\times 3}\right)$$

$a_i \sim N(0,1), i = 1, \ldots, r$
$\sigma^2 \sim logNormal(0,1),$
$\phi \sim N(0, 0.1), \theta \sim N(0, 0.1), \psi \sim N(0, 0.1)$
$t_x \sim N(0,5), t_y \sim N(0,5), t_z \sim N(0,5)$

Pose parameters

Giving up correspondence



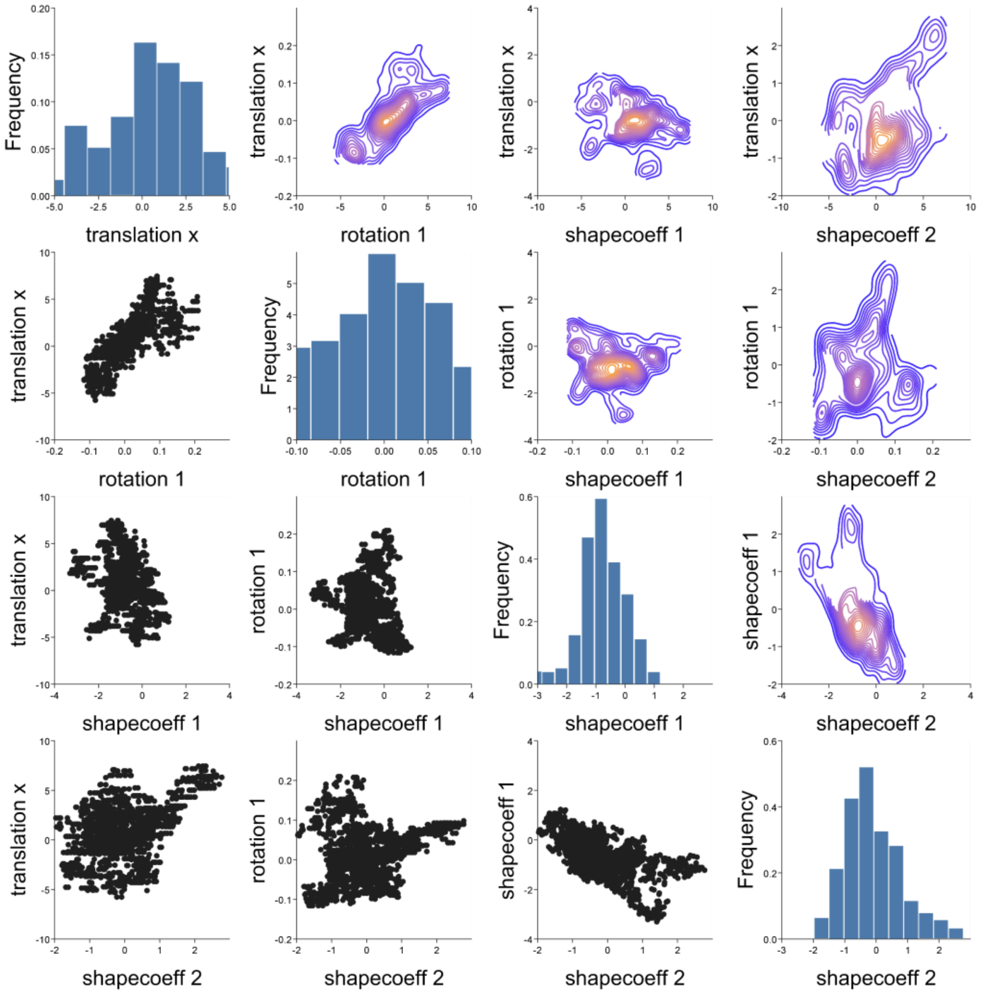
Model

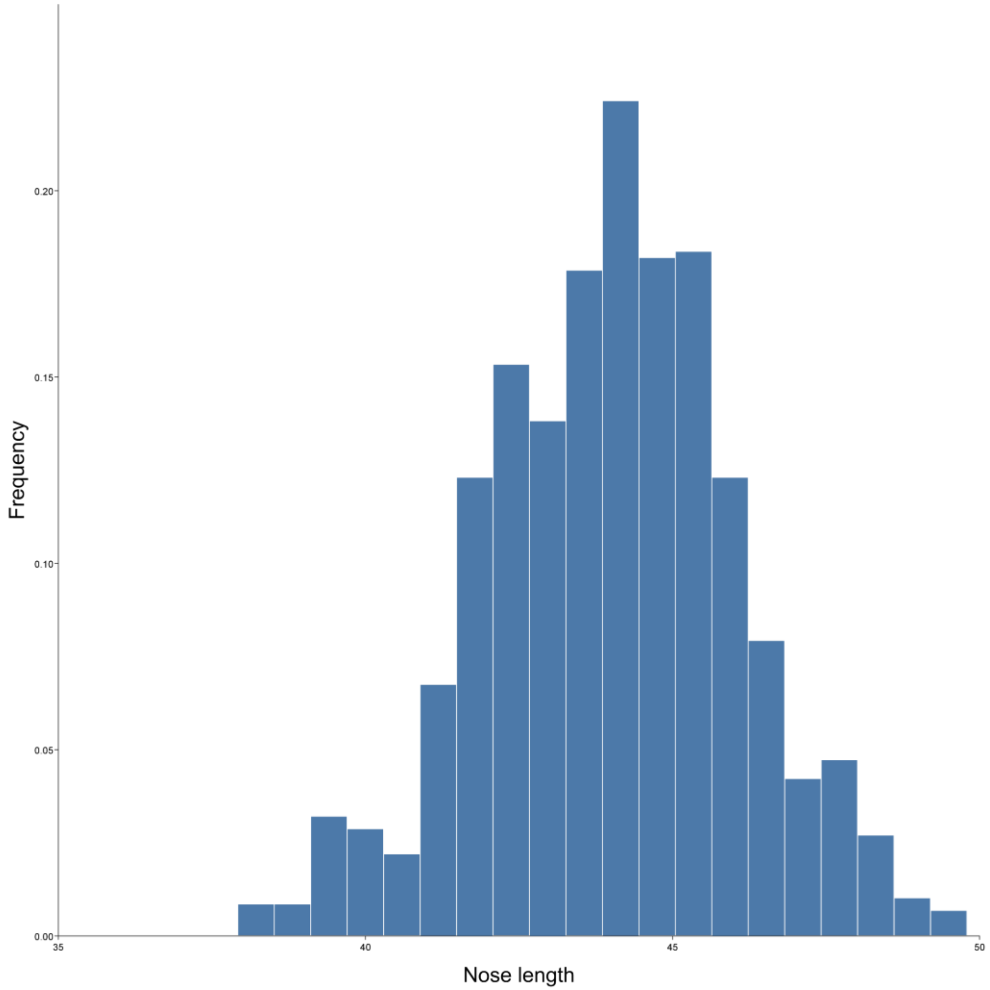
$$p\left(x_i^T \middle| \alpha_1 \dots \alpha_n, \sigma^2, \phi, \theta, \psi, t_x, t_y, t_z\right) = N\left(ClosestPoint(x_i^T, \Gamma_R[\alpha, \phi, \theta, \psi, t]), \sigma^2 I_{3\times3}\right)$$

$$a_i \sim N(0,1), i = 1, \dots, r$$
$$\sigma^2 \sim logNormal(0,1)$$
$$\phi \sim N(0,0.1), \theta \sim N(0,0.1), \psi \sim N(0,0.1)$$
$$t_x \sim N(0,5), t_y \sim N(0,5), t_z \sim N(0,5)$$

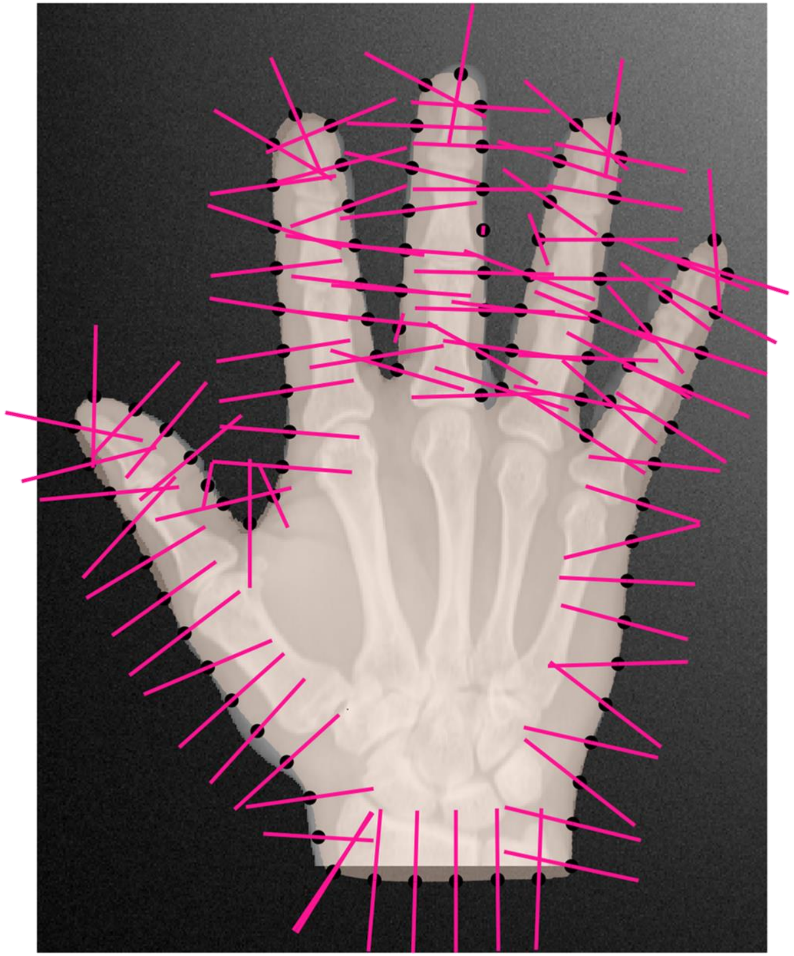
Surface, generated by model parameters $\alpha, \phi, \theta, \psi, t$

Posterior distribution





Beyond fitting shape



Model

Likelihood : ???

Priors:

$a_i \sim N(0,1), i = 1, \dots, r$
$\sigma^2 \sim logNormal(0,1)$
$\phi \sim N(0, 0.1), \theta \sim N(0, 0.1), \psi \sim N(0, 0.1)$
$t_x \sim N(0,5), t_y \sim N(0,5), t_z \sim N(0,5)$

Works in principle for any type of data, but …
… the more complicated the model and synthesis
function, the harder the fitting problem.