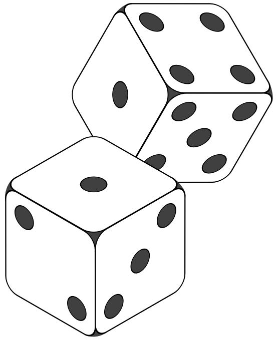
University of Basel

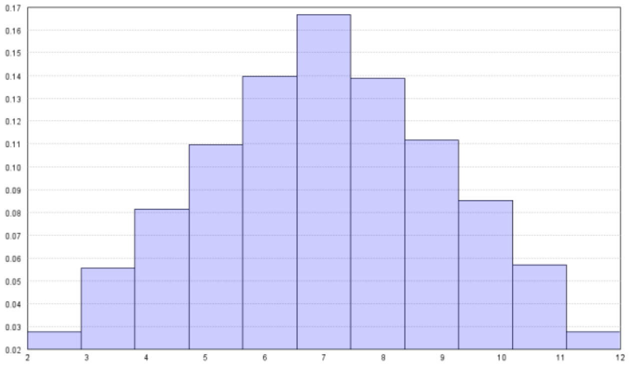
Bayesian probability

Marcel Lüthi, Departement of Mathematics and Computer Science, University of Basel

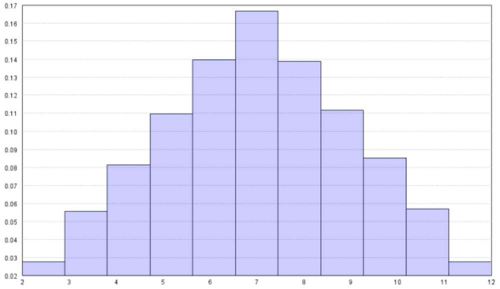
Different interpretations of probabilities

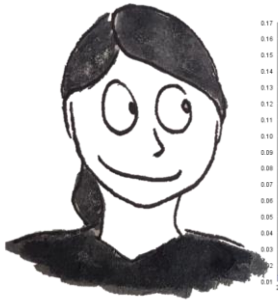
Long-term frequencies

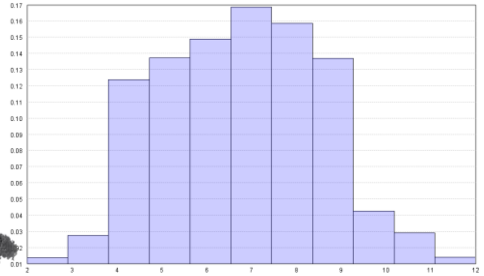




Degree of belief (Bayesian probability)







Bayesian probability in shape and image analysis

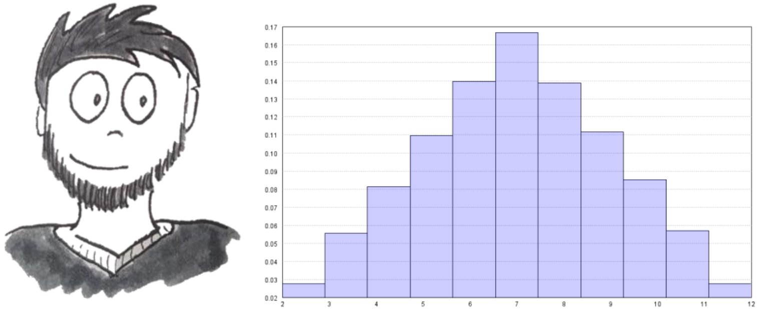
Sources of uncertainty in image and shape analysis

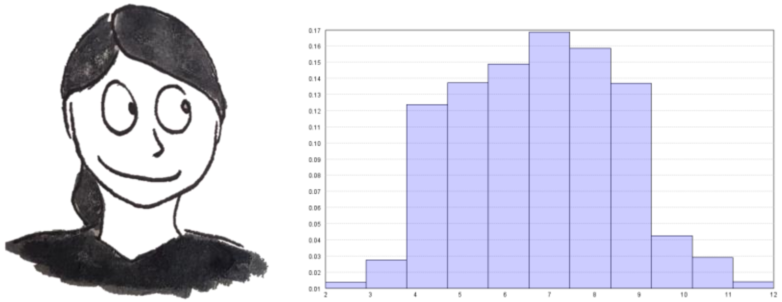
- Measurement noise
- Calibration of acquisition device
- Limited measurement accuracy
- Missing data

Repeating a measurement does not give us much more information.

Bayesian probability

Bayesian probabilities rely on a *subjective* perspective:

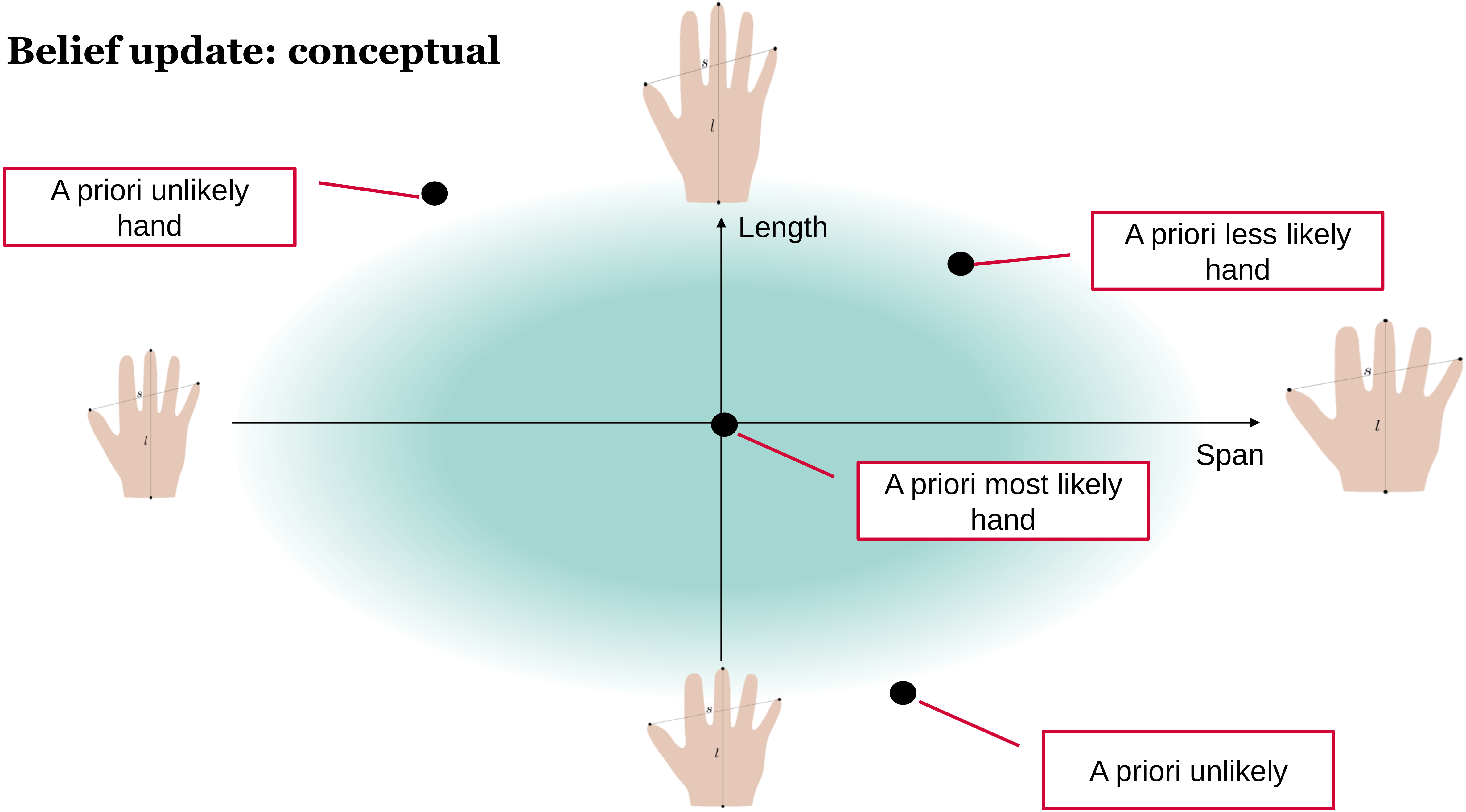




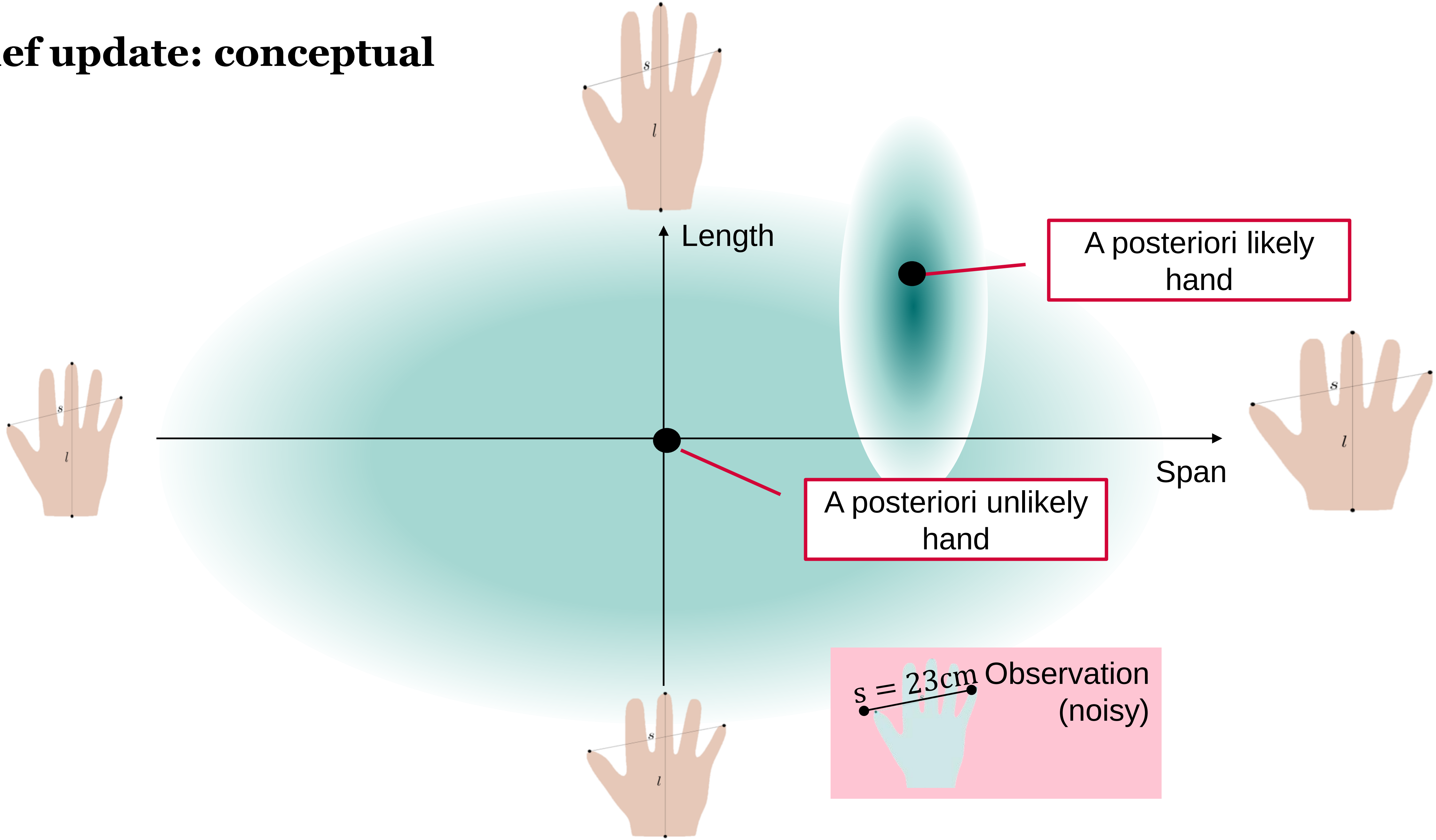
Subjective is not arbitrary!

- Belief are updated when new data becomes available
- Belief update follow rules of probability theory.

Belief update: conceptual



Belief update: conceptual



Belief update: Conditioning and Bayes Rule

Starting point: Joint distribution (usually factorized)

$$p(s, l) = p(s|l)p(l)$$

Example:

1. *Length of an average hand is approximately 24 centimeter*
   ca 2/3 of the hands are within a range of ± 2 *cm*
   $$p(l) = N(24, 2)$$

2. *On average, length and span are approximately the same*
   The span varies more than the length. 2/3 of the hands are within a 4 cm range
   $$p(s|l) = N(l, 4)$$

Belief update: Conditioning and Bayes Rule

Starting point: Joint distribution (usually factorized)

$$p(s,l) = p(s|l)p(l)$$

Belief update: Conditioning

$$p(l|s) = \frac{p(s,l)}{p(s)} = \frac{p(s|l)p(l)}{p(s)} = \frac{p(s|l)p(l)}{\int p(s|l)p(l)dl}$$

Belief update is just conditioning and marginalizing!

Bayes rule

Rule for updating our belief in X, after we have observed data Y

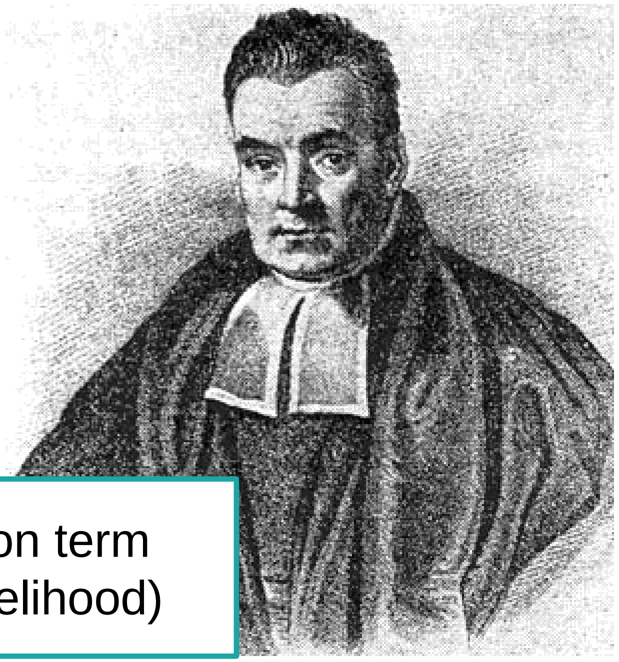
Likelihood

Prior

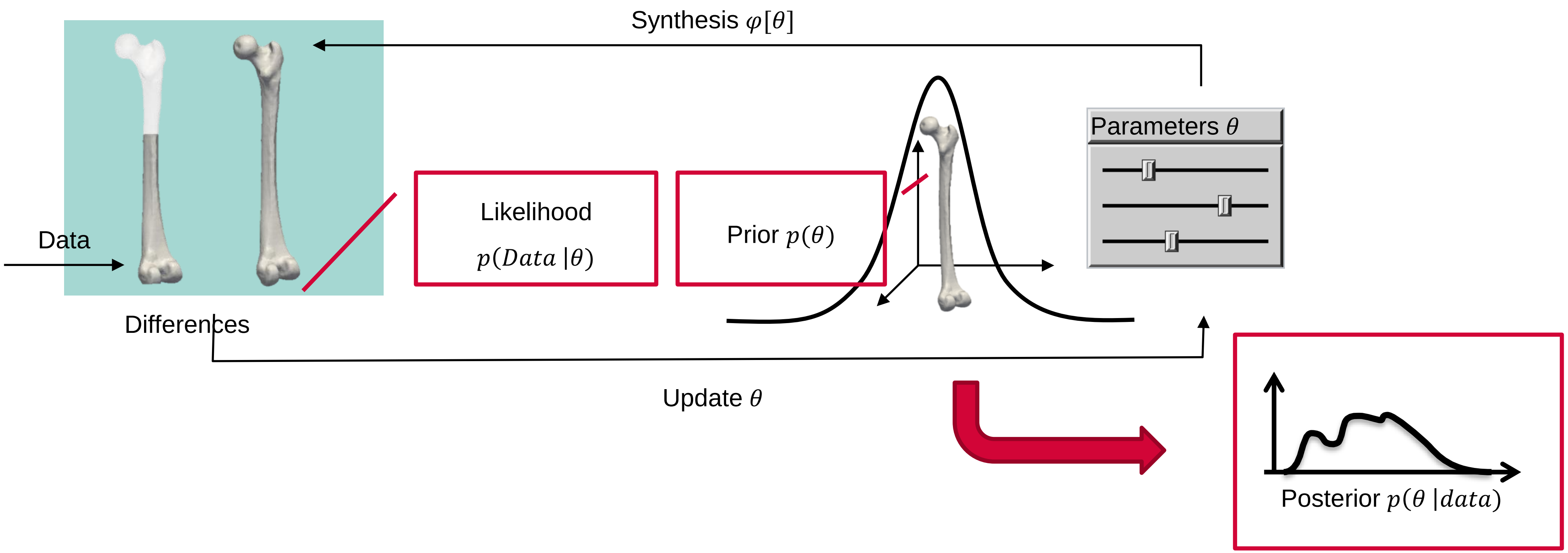
$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)} = \frac{P(Y|X)P(X)}{\int_X P(X)P(Y|X)}$$

Posterior

Normalization term
(marginal likelihood)



Bayesian Inference



Bayes rule

$$p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)} = \frac{p(D|\theta)p(\theta)}{\int p(D|\theta)p(\theta)d\theta}$$