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Analysis by synthesis

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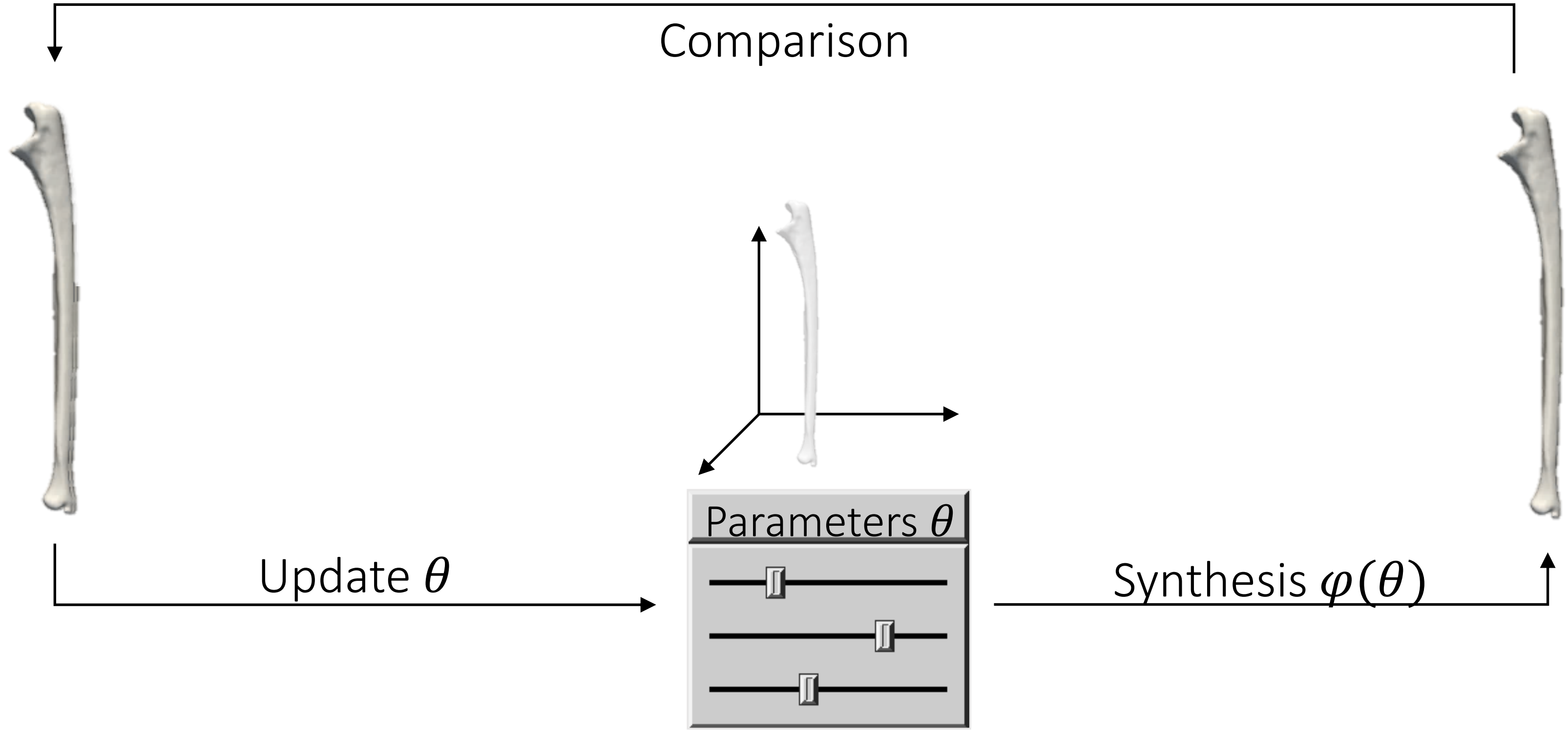
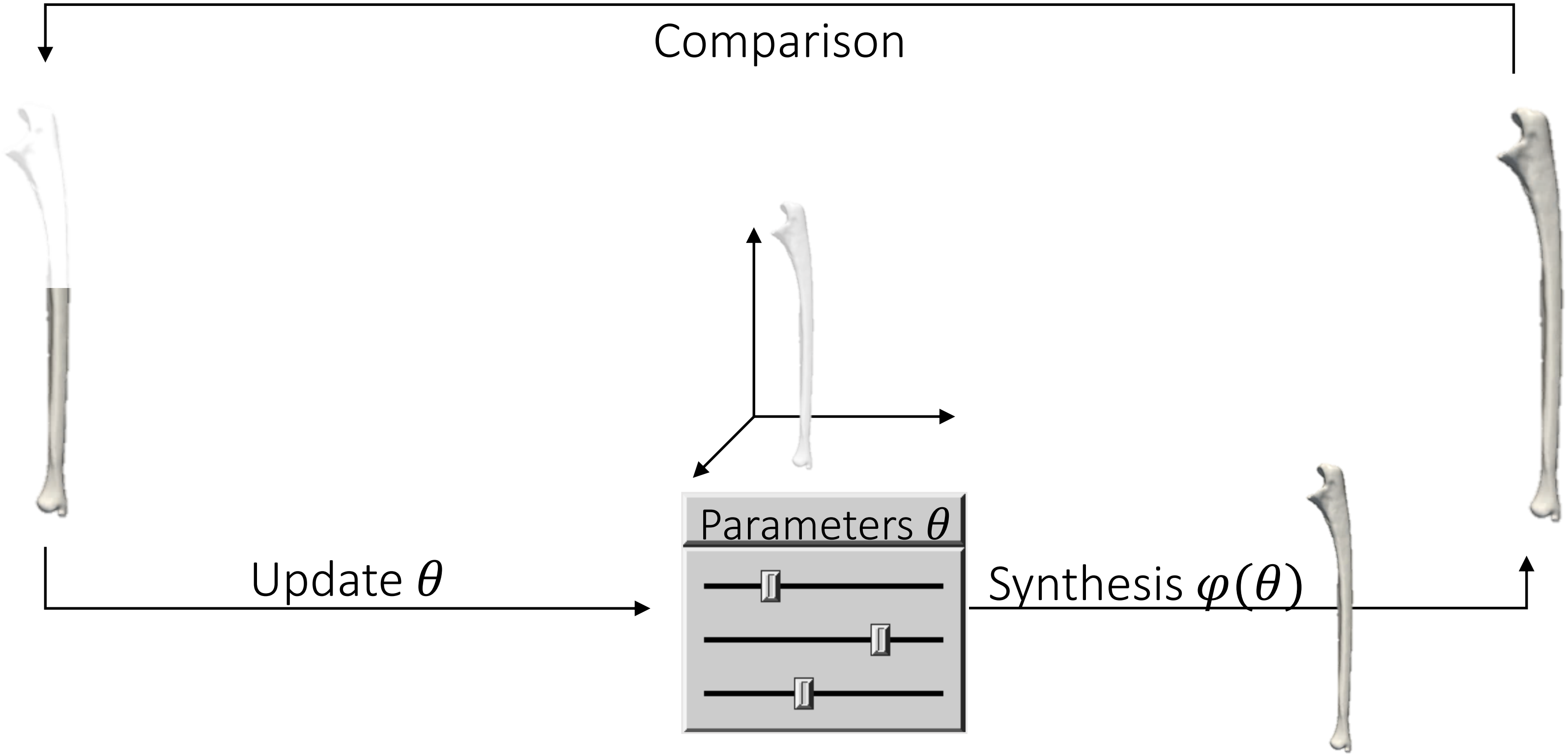
Analysis by synthesis



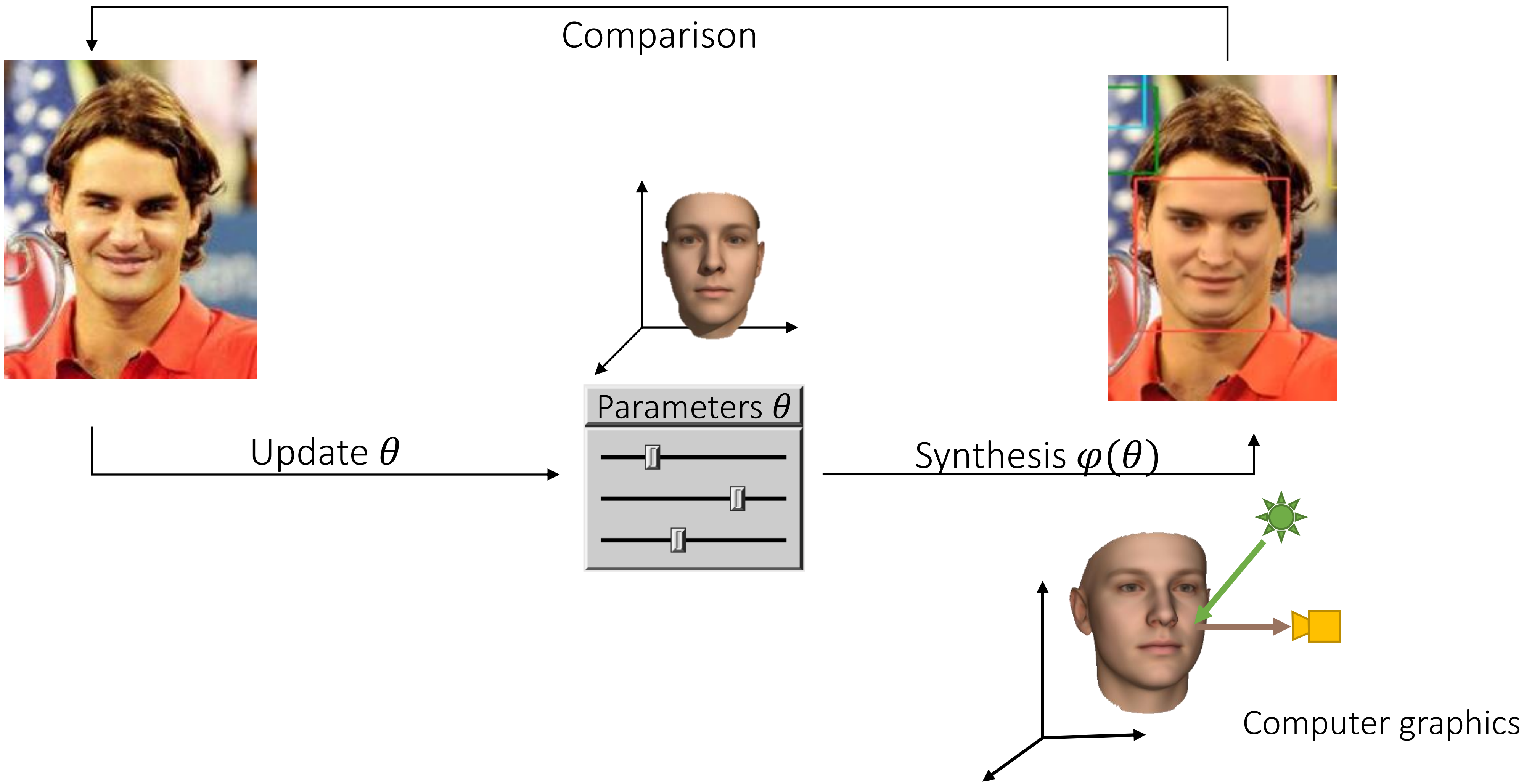
Being able to synthesize data means we can understand how it was formed.

– Allows reasoning about unseen parts.

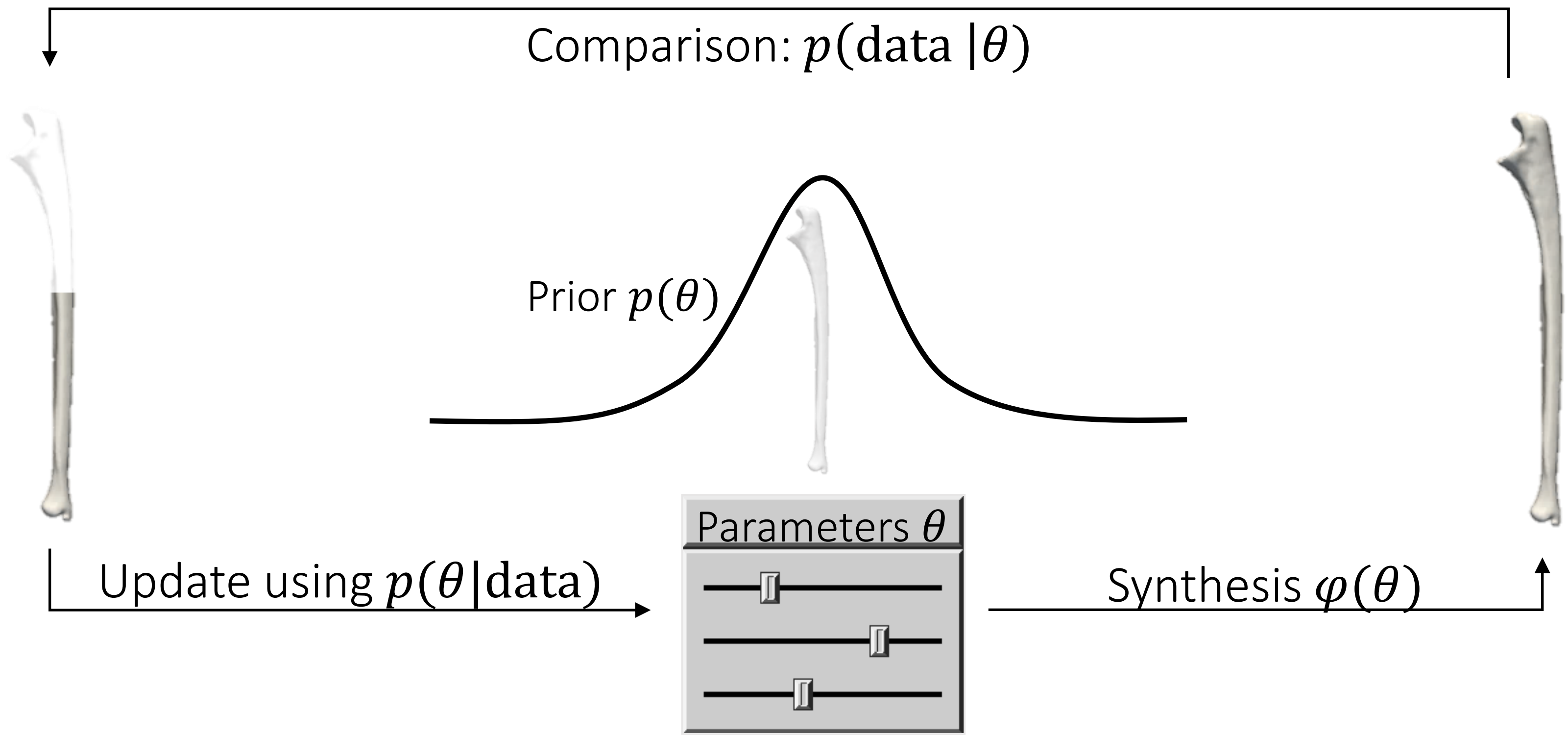
Analysis by synthesis



Analysis by synthesis – Computer vision

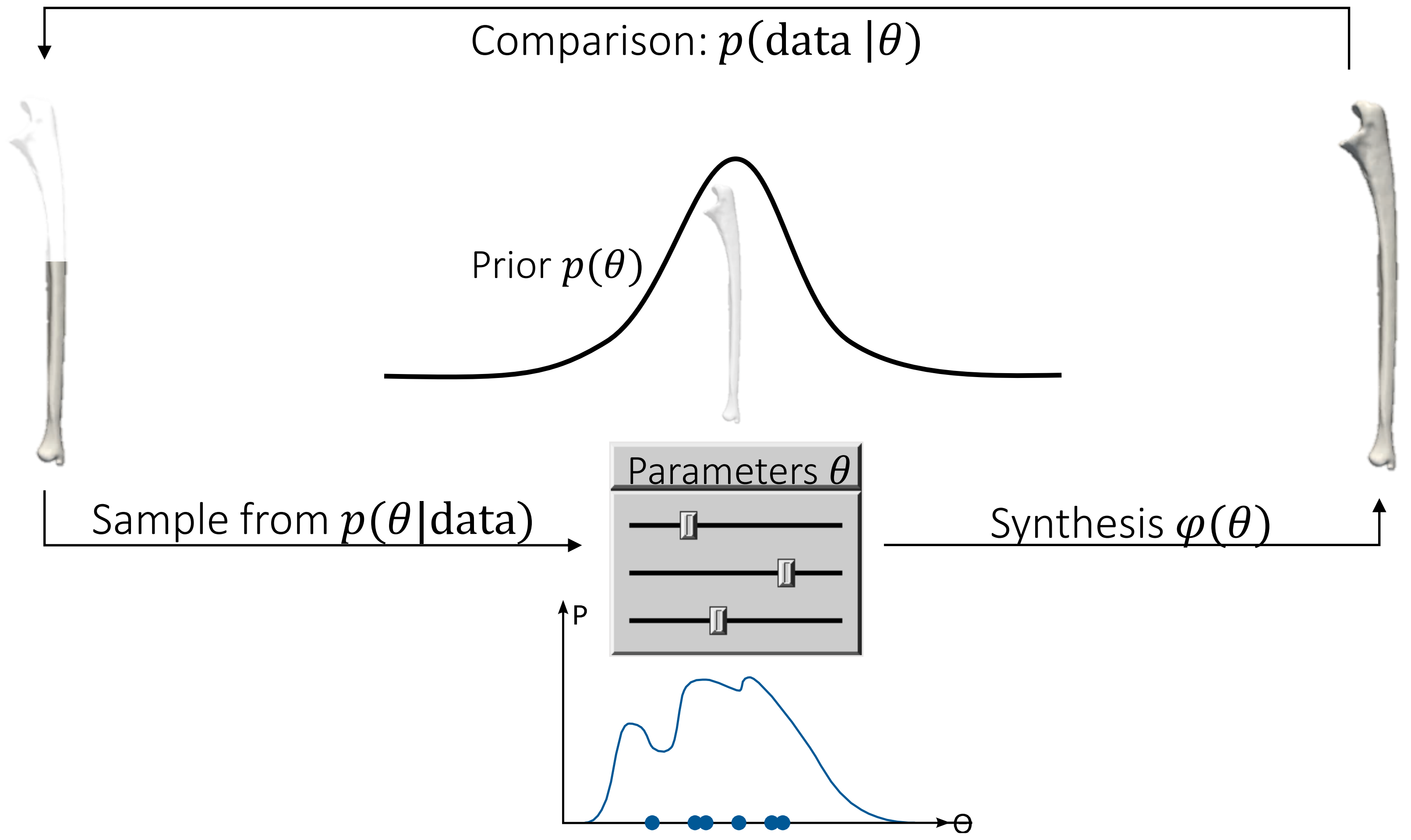


Mathematical Framework: Bayesian inference



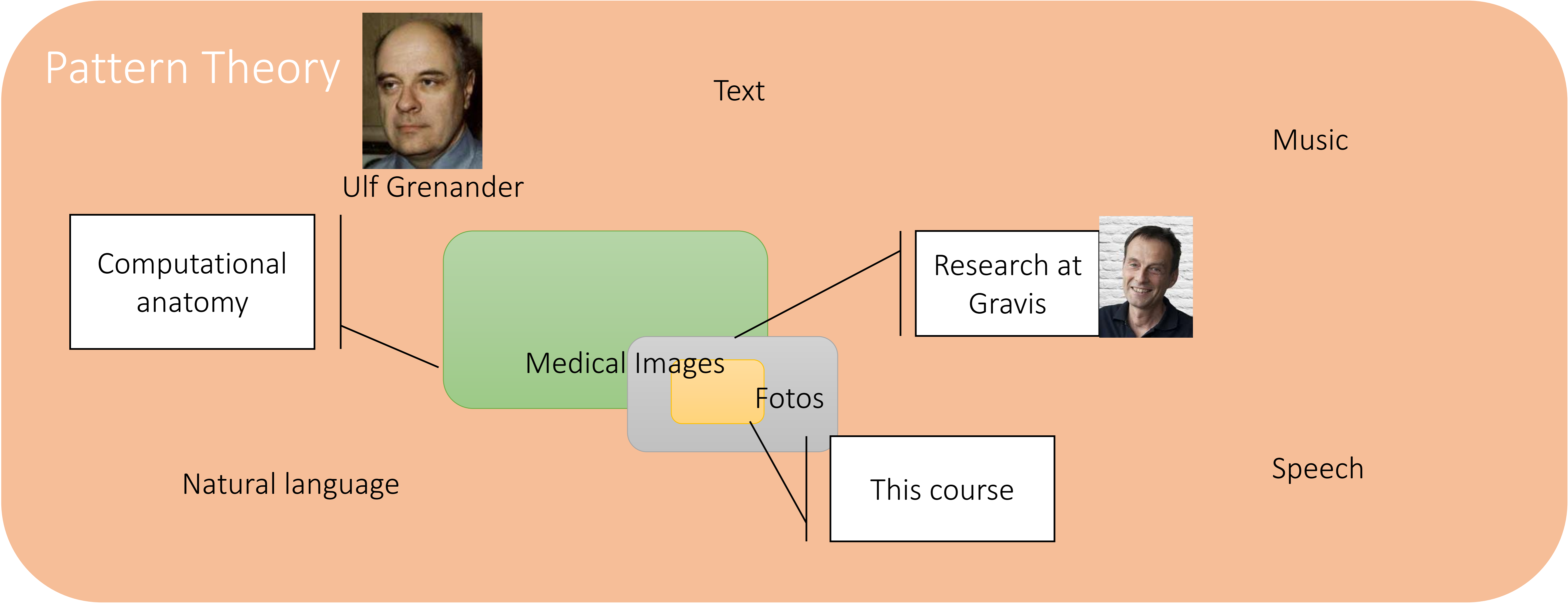
- Principled way of dealing with uncertainty.

Algorithmic implementation: MCMC

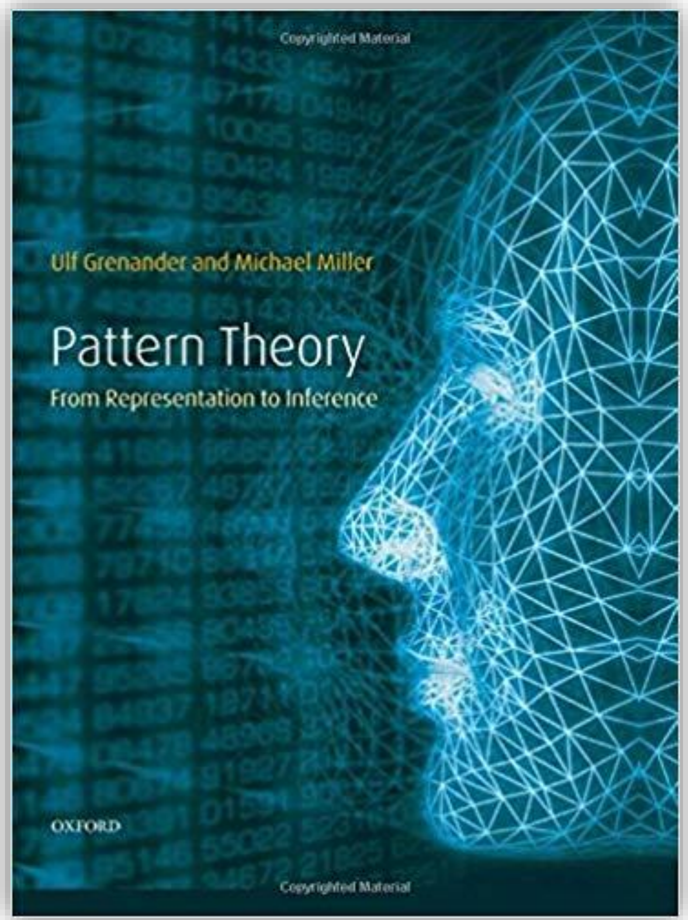


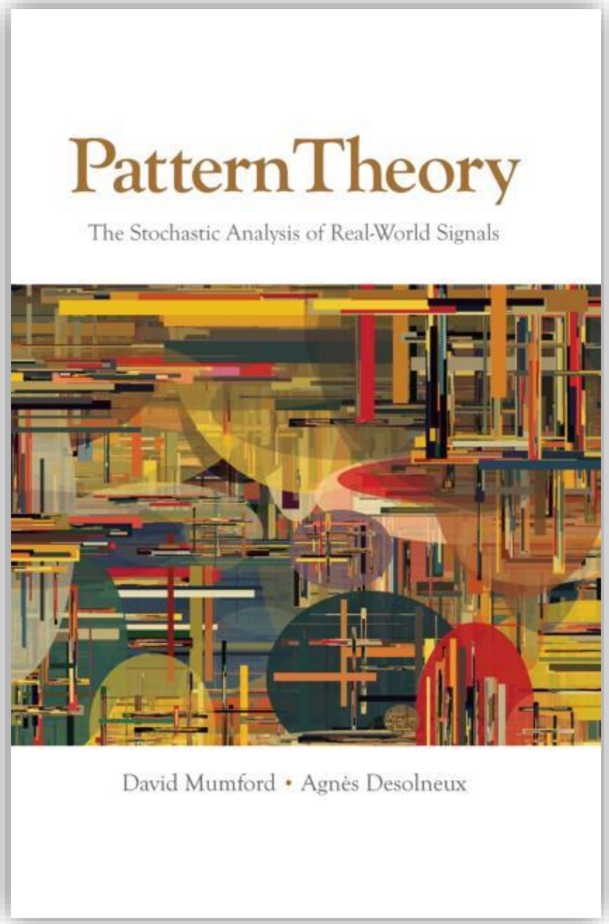
Posterior distribution over parameters

The course in context



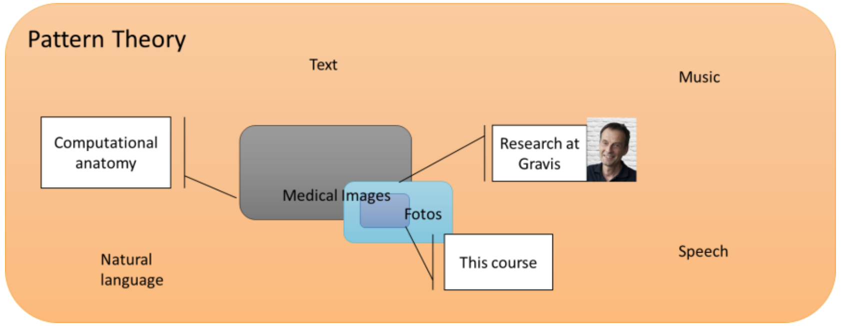
Pattern theory – The mathematics





Pattern theory vs PMM

- Pattern theory is about **developing** a theory for understanding real-world signals
- Probabilistic Morphable Models are about **using** theoretical well founded concepts to analyse images.
  - GPs for modelling
  - MCMC for model fitting
  - Working software

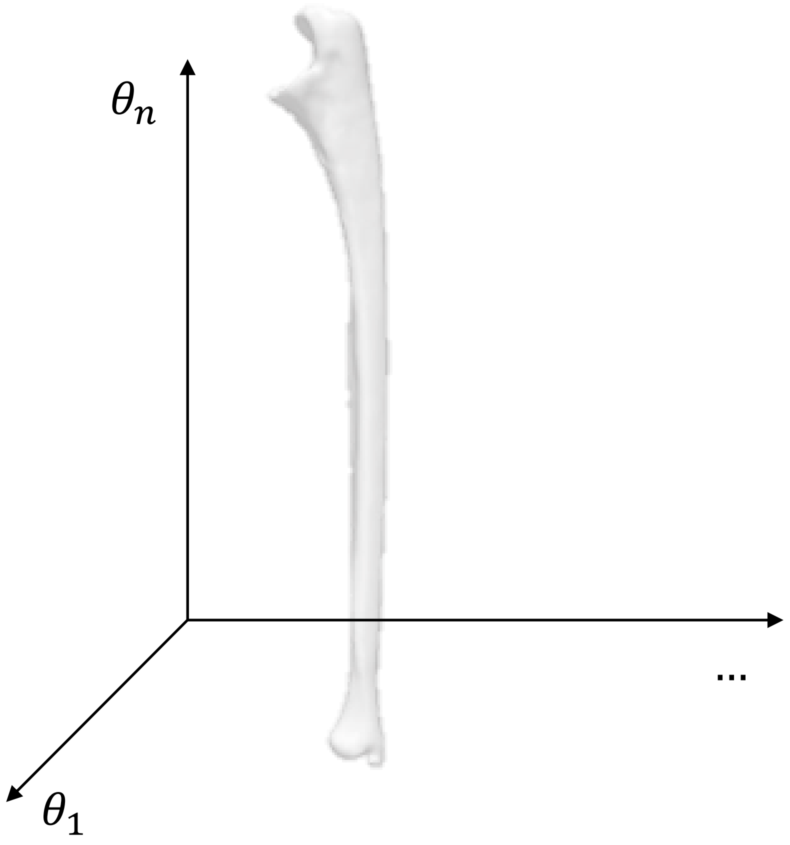


Analysis by Synthesis in 5 (simple) steps

Analysis by synthesis in 5 simple steps

1. Define a parametric model

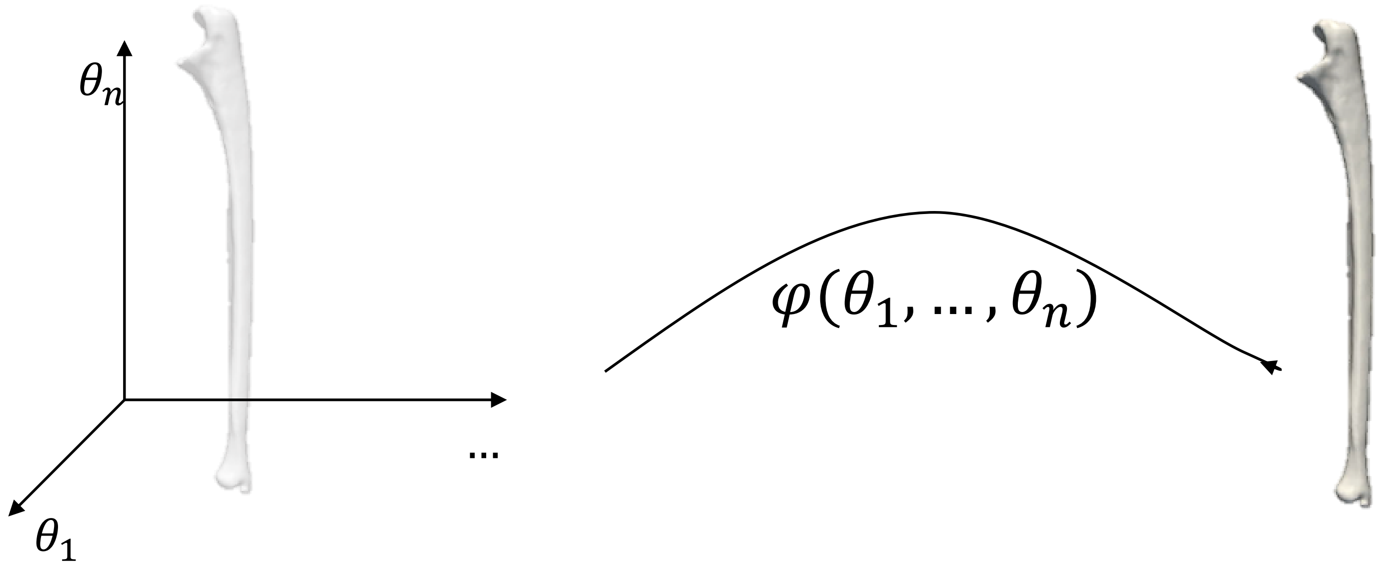
- a representation of the world
- State of the world is determined by parameters $\theta = (\theta_1, \ldots, \theta_n)$



Analysis by synthesis in 5 simple steps

2. Define a synthesis function $\varphi(\theta_1, \dots, \theta_n)$

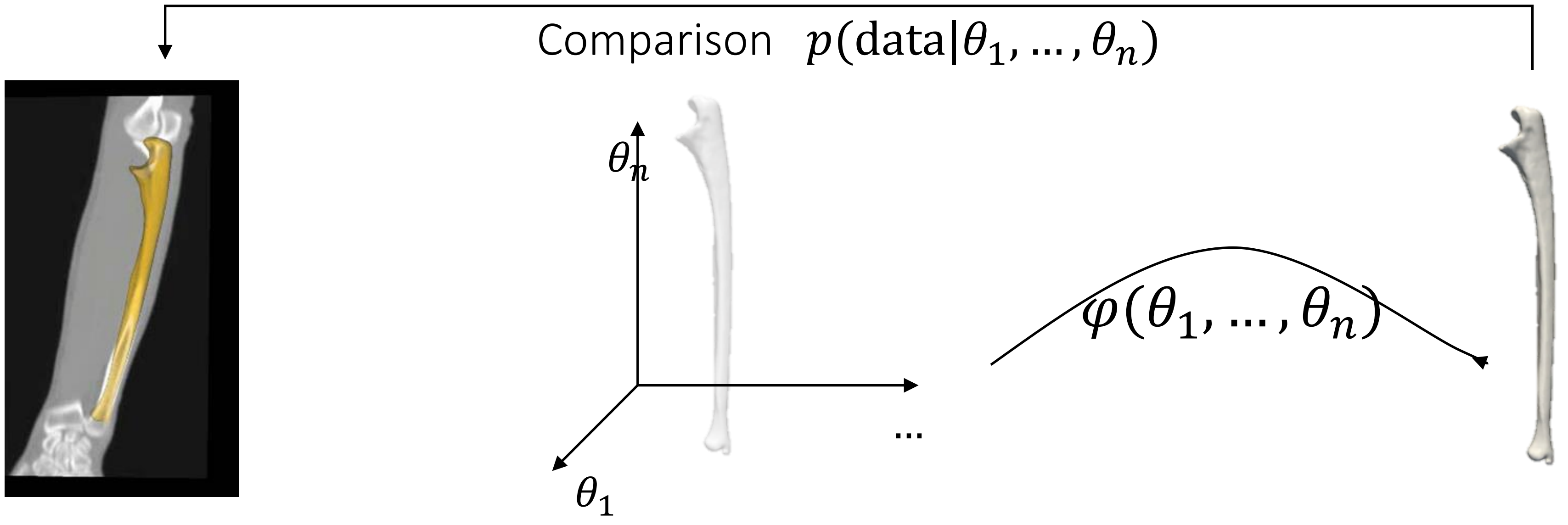
- generates/synthesize the data given the "state of the world"
- φ can be deterministic or stochastic



Analysis-by-Synthesis in 5 simple steps

3. Define likelihood function:

- Define a probabilistic model $p(\text{data}|\theta_1, \ldots, \theta_n)$ that models how the synthesized data compares to the real data
- Includes stochastic factors on the data, such as noise



Bayesian inference

We have: $P(data|\theta_1, \dots, \theta_n)$
We want: $P(\theta_1, \dots, \theta_n|data)$

Bayes rule:

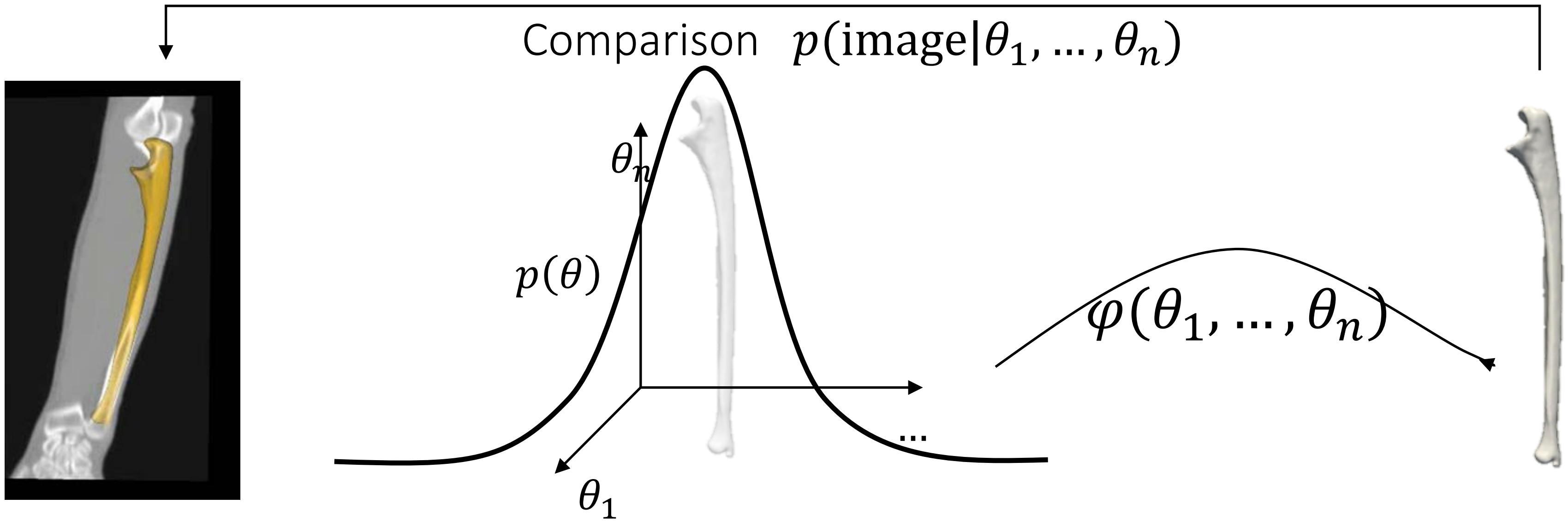
$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)}$$

Lets us compute from $p(D|\theta)$ *its "inverse"* $p(\theta|D)$

Analysis by synthesis in 5 simple steps

4. Define prior distribution: $p(\theta) = p(\theta_1, \dots, \theta_n)$
   - Our believe about the "state of the world"

- Makes it possible to invert mapping $p(\text{data}|\theta_1, \dots, \theta_n)$



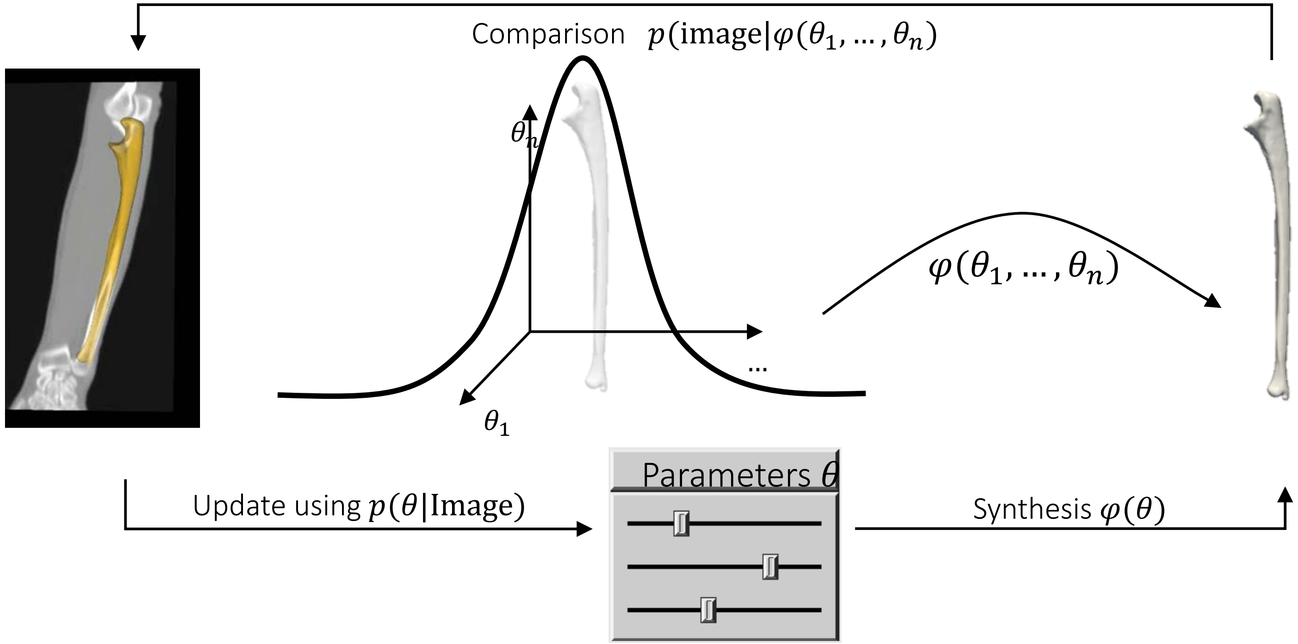
Analysis by synthesis in 5 simple steps

5. Do inference

$$p(\theta_1, \dots, \theta_n | \text{data}) = \frac{p(\theta_1, \dots, \theta_n) p(\text{data} | \theta_1, \dots, \theta_n)}{p(\text{data})}$$

Purely conceptual formulation:

- Independent of algorithmic implementation
- But usually done iteratively



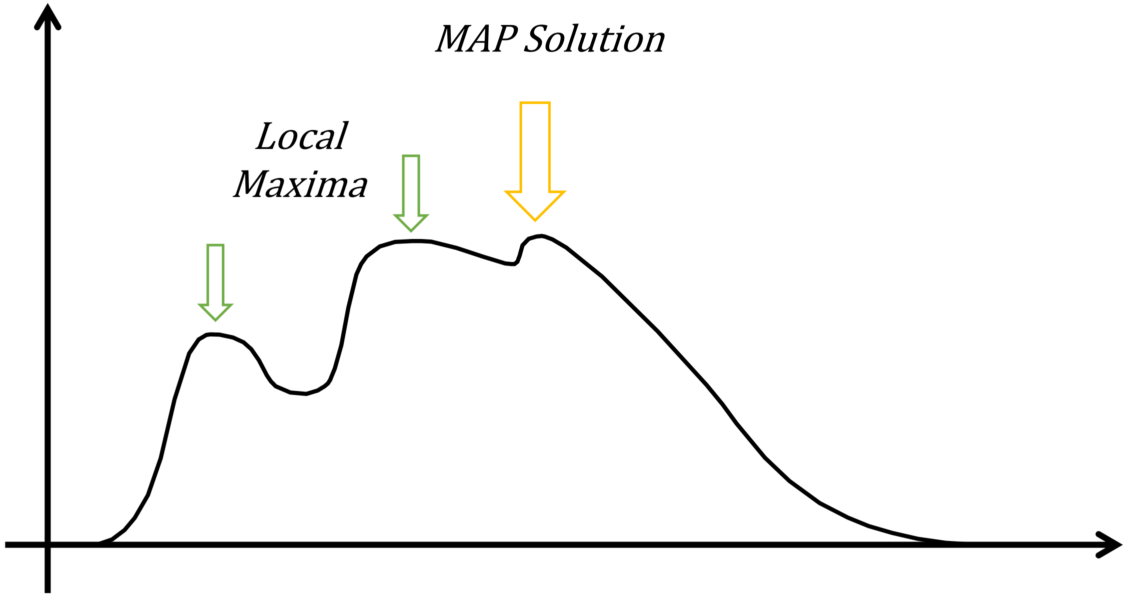
Analysis by synthesis in 5 simple steps

5. Possibility 1: Find best (most likely) solution:

$$\arg\max_{\theta_1,\ldots,\theta_n} p(\theta_1, \ldots, \theta_n | \text{data}) = \arg\max_{\theta_1,\ldots,\theta_n} \frac{p(\theta_1, \ldots, \theta_n) p(\text{data} | \theta_1, \ldots, \theta_n)}{p(\text{data})}$$

Most popular approach

- Usually based on gradient-descent
- May miss good solutions



Analysis by synthesis in 5 simple steps

5. Possibility 2: Find posterior distribution:

$$p(\theta_1, \dots, \theta_n|\text{data}) = \frac{p(\theta_1, \dots, \theta_n)p(\text{data}|\theta_1, \dots, \theta_n)}{p(\text{data})}$$

Core of this course

- Obtain samples from the distribution
- Based on Markov Chain Monte Carlo methods