Summer school on Probabilistic Morphable Models

Basel, 25. June – 29. June

Probabilistic Shape Modelling
- Analysis by Synthesis -

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University of Basel

Probabilistic Morphable Models

Online Course / Futurelearn

Summer school

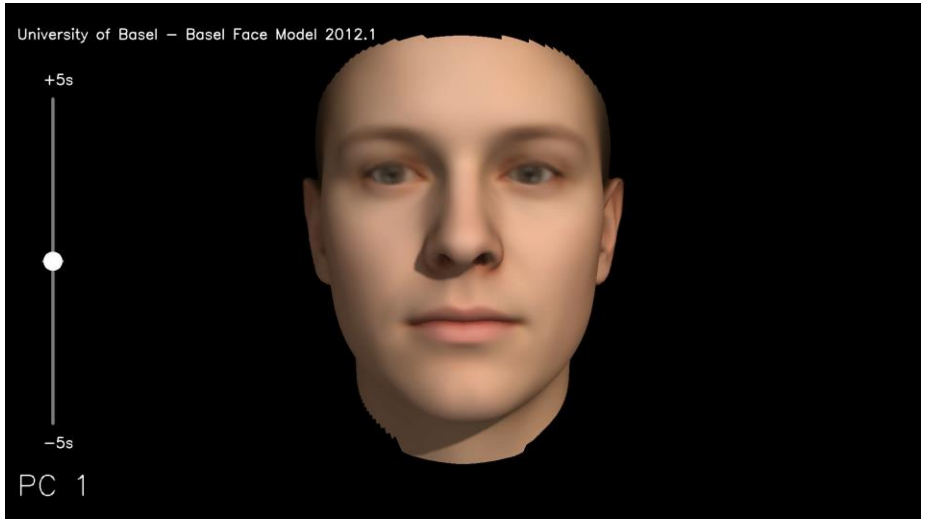
Shape Modelling

Model fitting

Scalismo

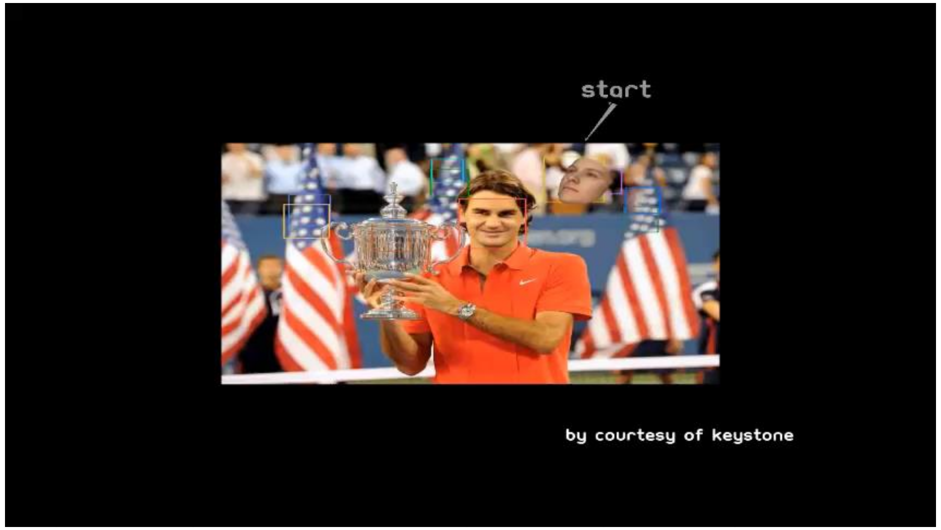
Probabilistic Morphable Models

Online Course / Futurelearn



Shape Modelling

Summer school



Model fitting

Scalismo

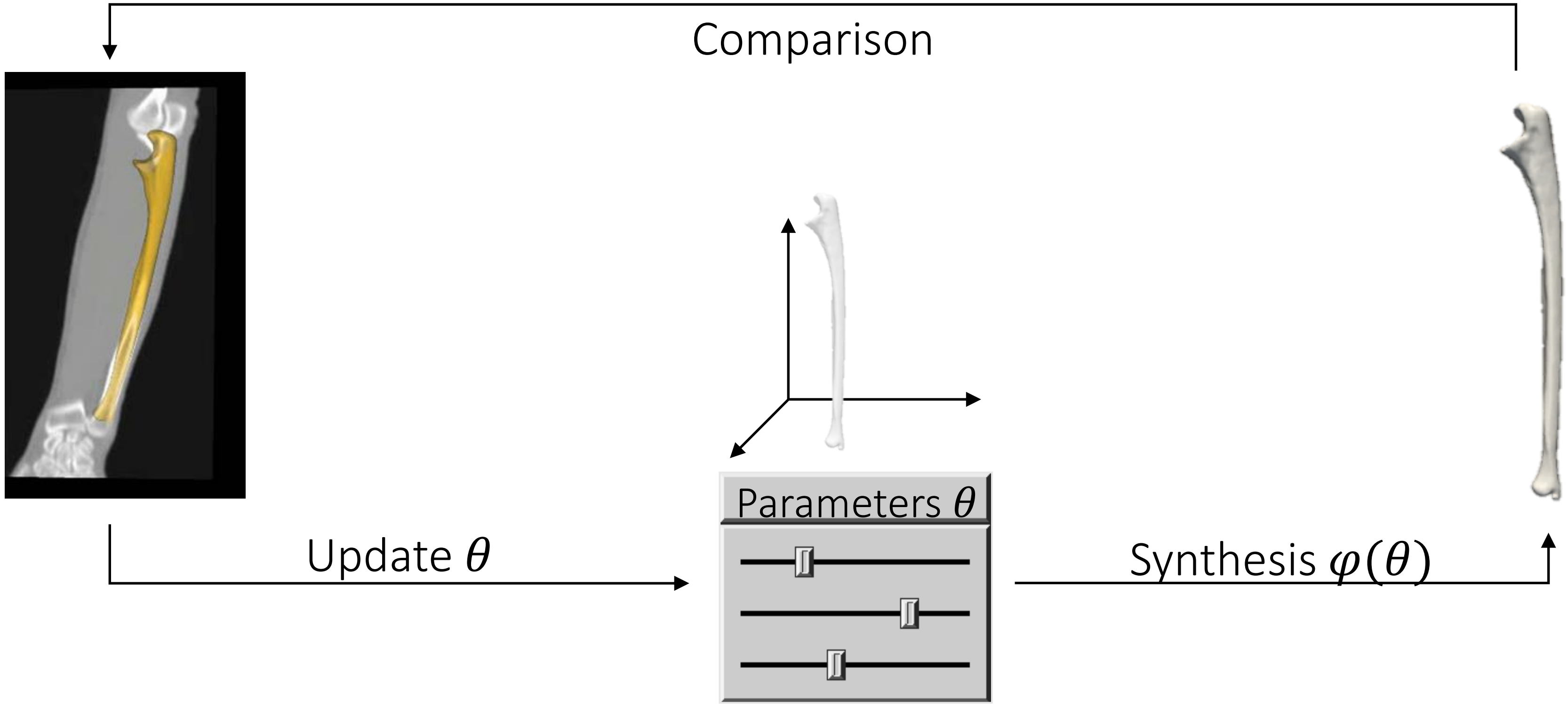
Programme

	Morning (09:00 – 12:00)	Afternoon (13:30 – 17:00)
Monday	• Analysis by Synthesis • Non-rigid registration • Excercise: Registration in Scalismo	• Spotlight presentations • Course project - Setup • 17:00 - Welcome reception
Tuesday	• Bayesian inference • Markov Chain Monte Carlo – An algorithmic introduction • Exercises: MCMC Fitting in Scalismo	• Course project
Wednesday	• Introduction to Computer Graphics • 2D Face image analysis using MCMC	• Course project
Thursday	• Face image analysis – a look back • Understanding MCMC	• Course project
Friday	• Some Advanced topics • Project presentation	• Social event

Outline: First lecture

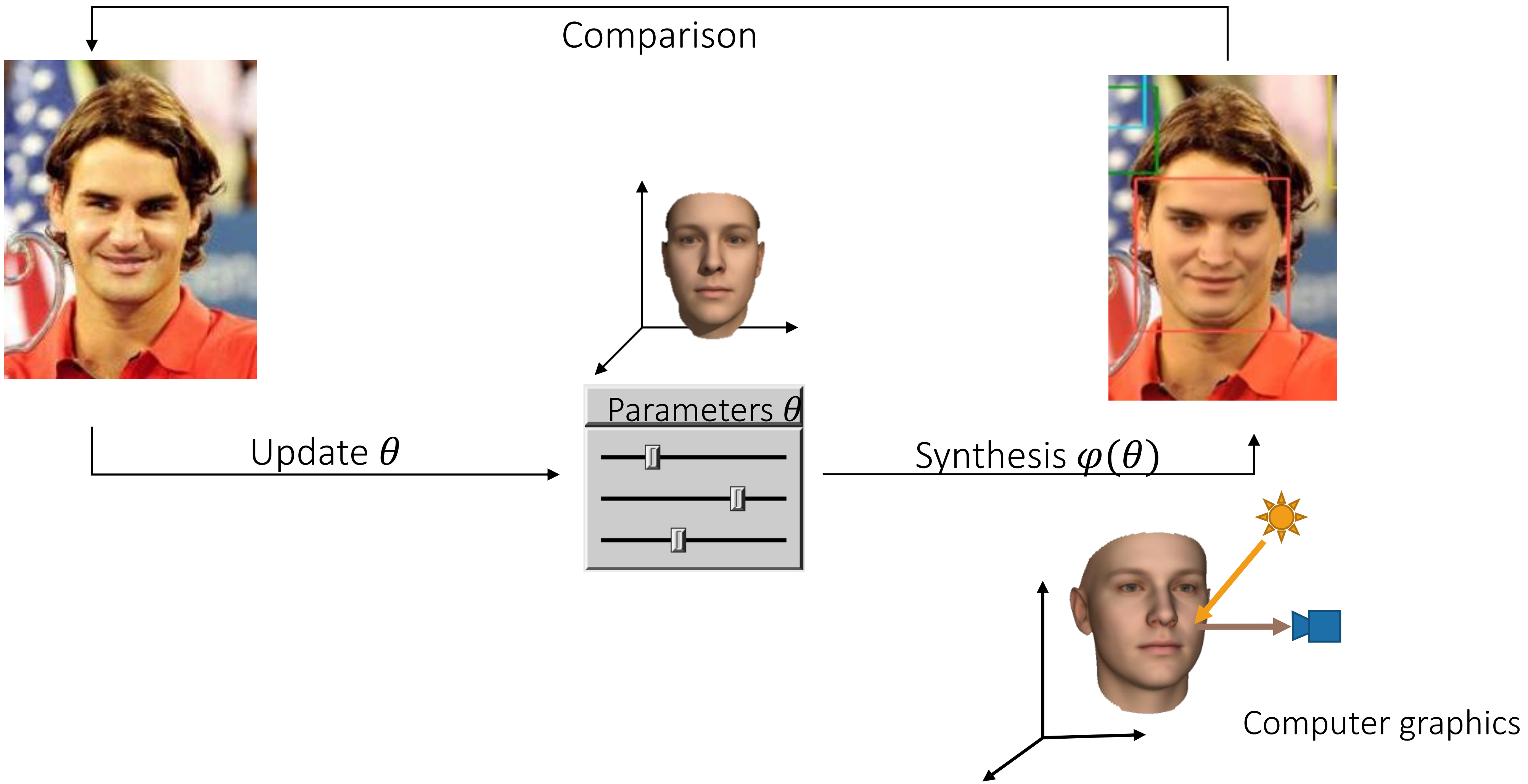
- Analysis by synthesis
  - The conceptual framework we follow in this course
- Computer vision verse medical image analysis
  - Some commonalities and differences of the two fields
- Analysis by Synthesis in 5 (simple) steps
  - One way how to implement the analysis by synthesis principle.

Conceptual Basis: Analysis by synthesis

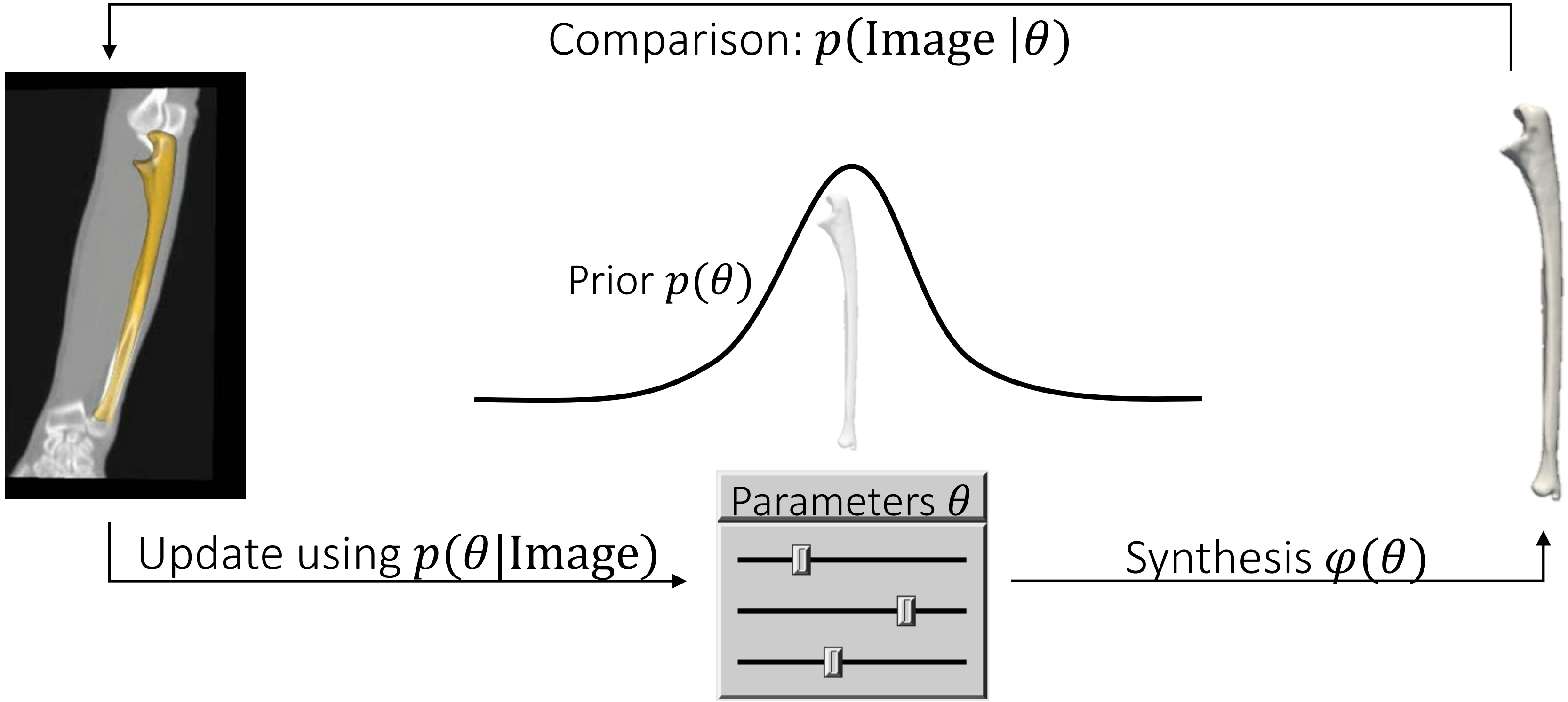


- If we are able to synthesize an image, we can explain it.
  - We can explain unseen parts and reason about them

Conceptual Basis: Analysis by synthesis

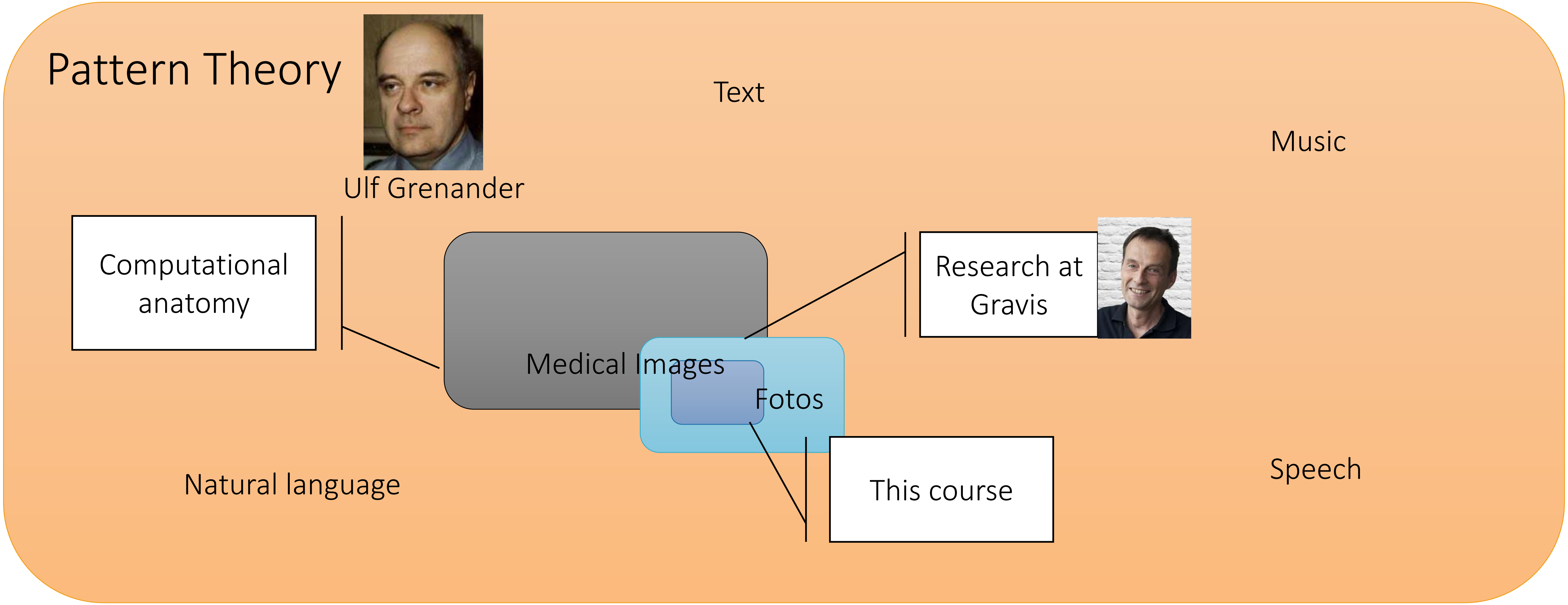


Mathematical Framework: Bayesian inference



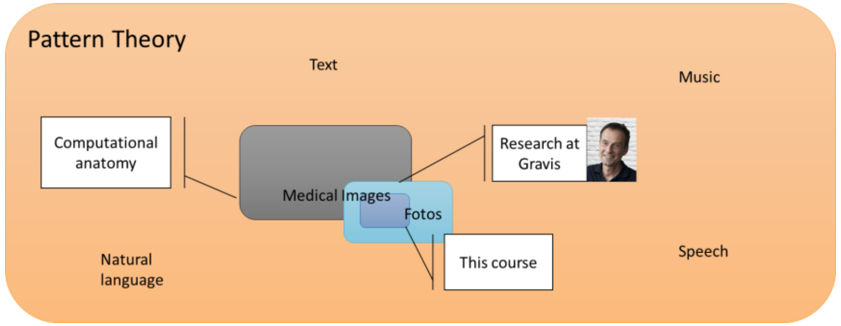
- Principled way of dealing with uncertainty.

The course in context



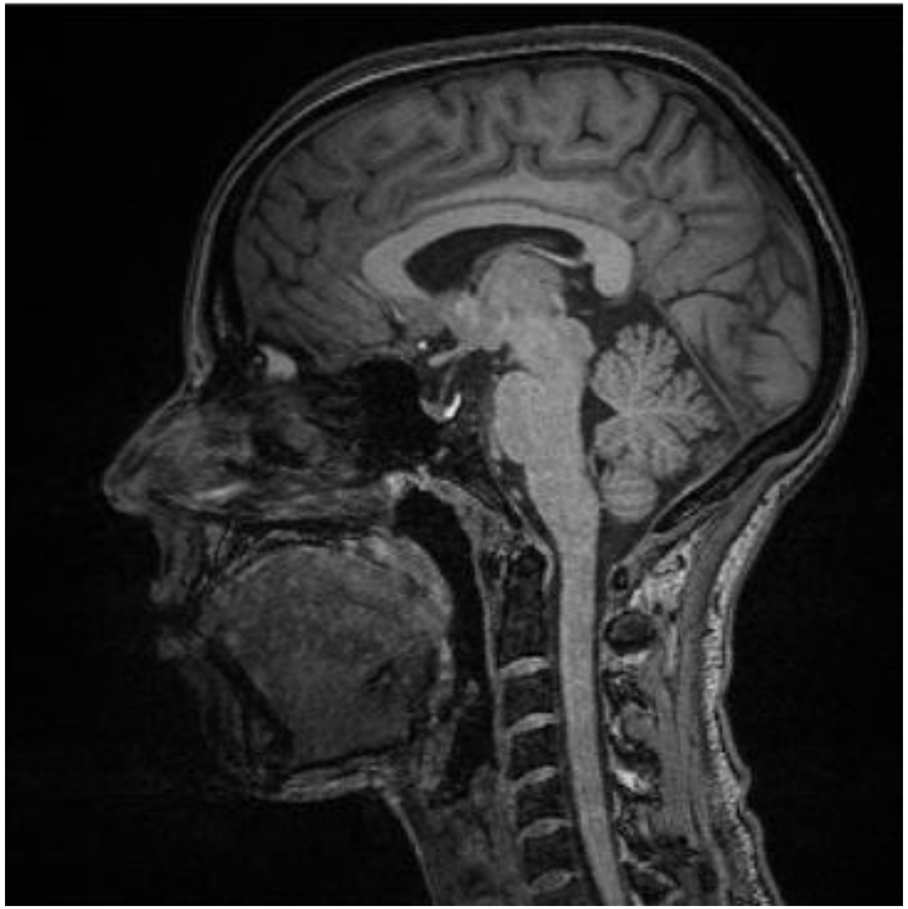
Pattern theory vs PMM

- Pattern theory is about **developing** a theory for understanding real-world signals
- Probabilistic Morphable Models are about **using** theoretical well founded concepts to analyse images.
  - GPs for modelling
  - MCMC for model fitting
  - Working software



Medical image analysis vs. Computer vision

Images: Medical Image Analysis vs Computer Vision



Source: OneYoungWorld.com

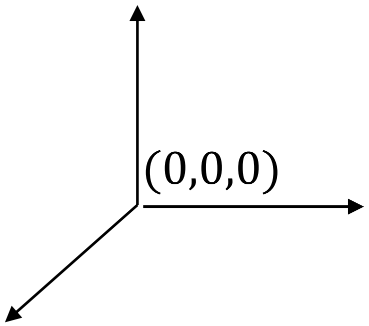
Images in medical image analysis

Goal: Measure and visualize the unseen

- Acquired with specific purpose
  - Controlled measurement
  - Done by experts
- Calibrated, specialized devices

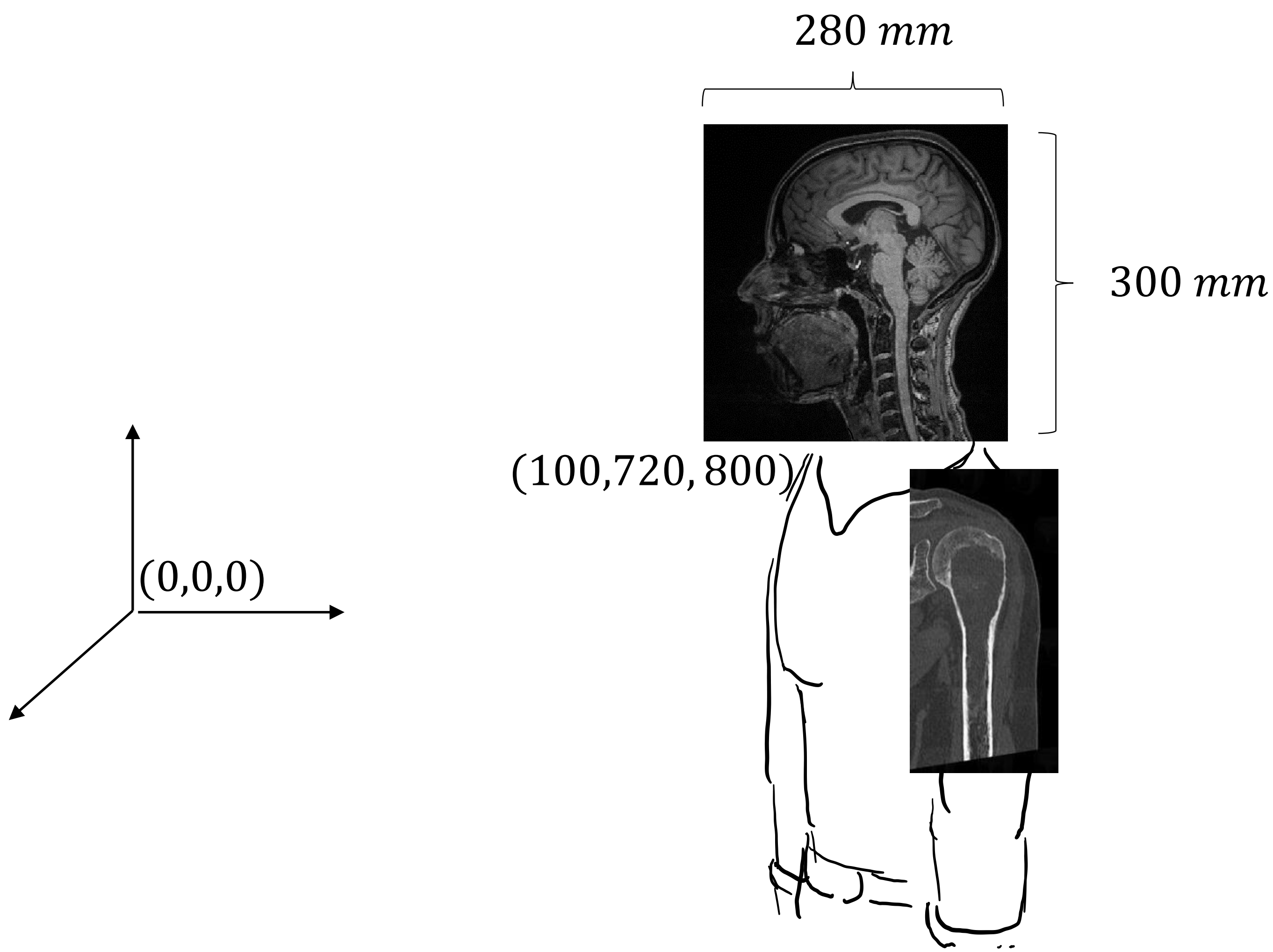
Source: www.siemens.com

Images in medical image analysis



- Images live in a coordinate system (units: mm)

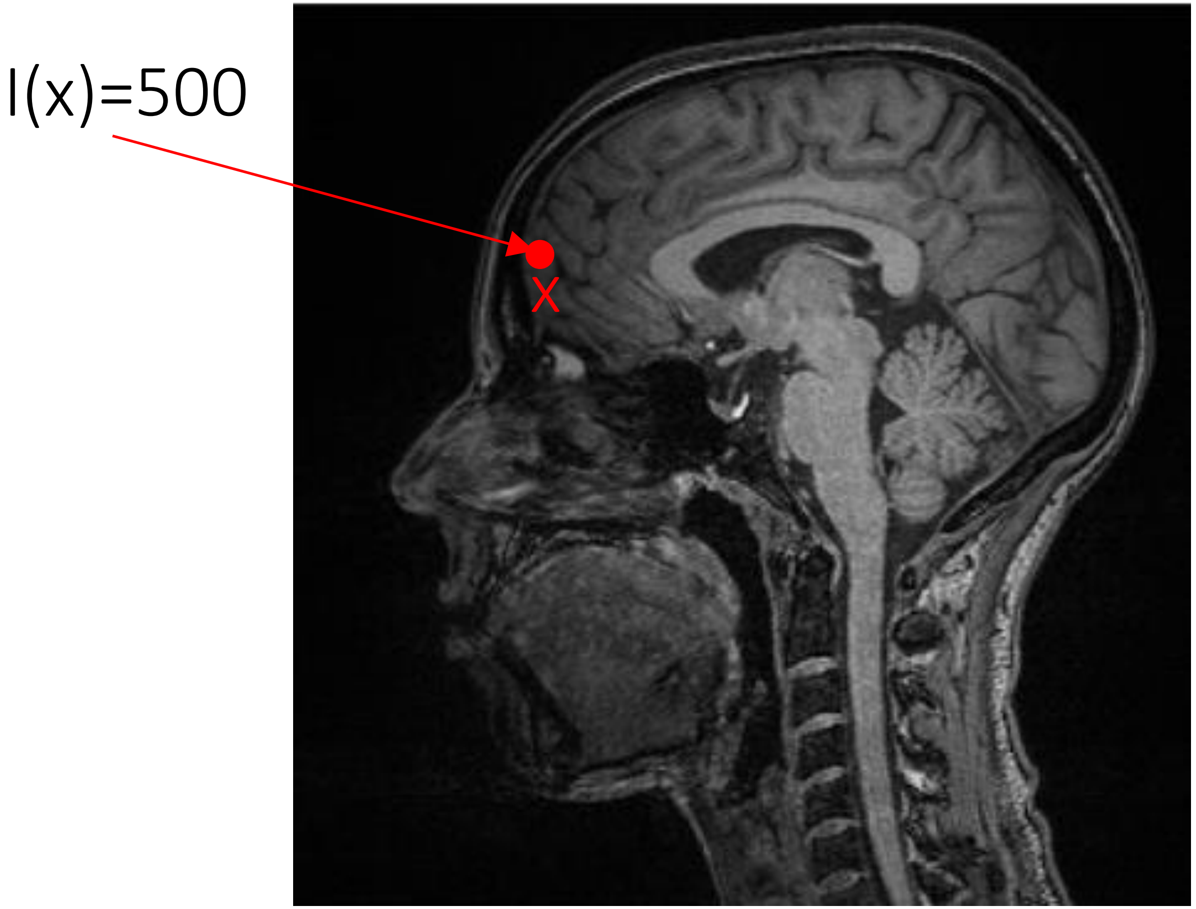
Images in medical image analysis



Images in medical image analysis

Values measure properties of the patient's tissue

- Usually scalar-valued
- Often calibrated
- CT Example:
  -1000 HU -> Air
  3000 HU -> cortical bone



Images in computer vision

Goal: Capture what we see in a realistic way

- Perspective projection from 3D object to 2D image
  - Many parts are occluded

Images in computer vision

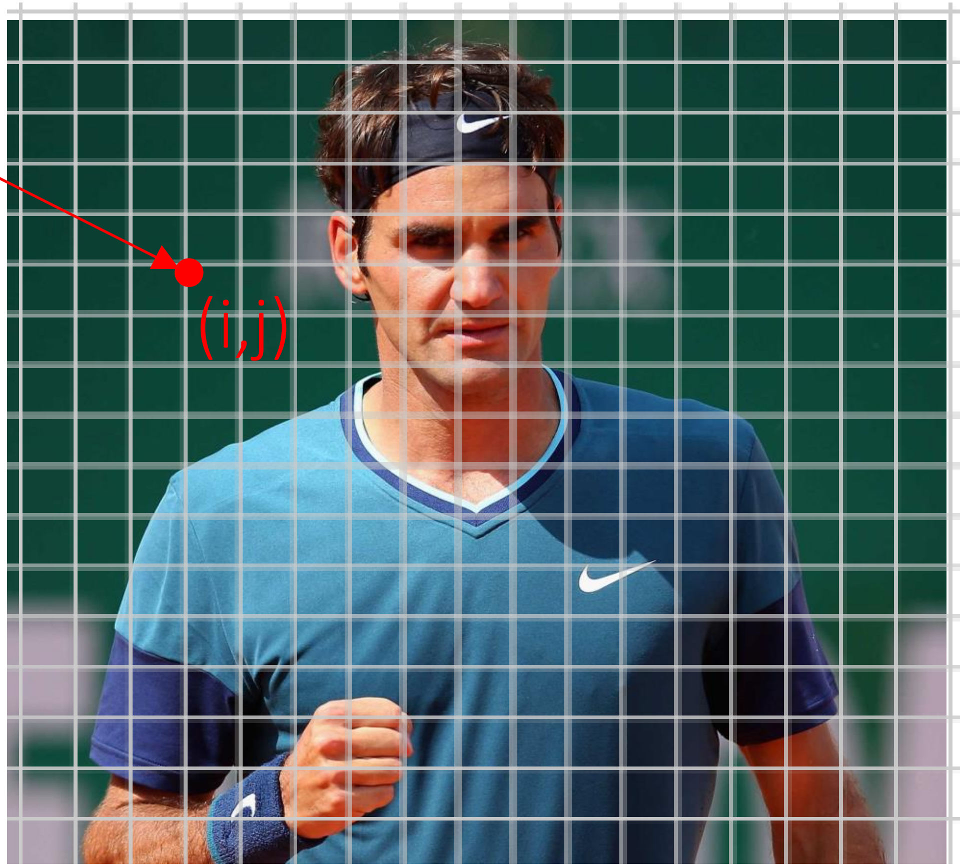
- Can be done by anybody
  - Acquisition device usually unknown
  - Uncontrolled background, lighting, …
- No clear scale
  - What is the camera distance?
- No natural coordinate system
  - Unit usually pixel

Source: twitter.com

Images in computer vision

- Pixels represent RGB values
- Values are measurement of light
  - Reproduce what the human eye would see
- Exact RGB value depends strongly on lighting conditions
  - Shadows
  - Ambient vs diffuse light

I(i,j)=(10,128, 2)



Images: Medical Image analysis vs Computer Vision

Medical image

- Controlled measurement
- Values have (often) clear interpretation
- Explicit setup to visualize unseen
- Coordinate system with clear scale

Computer vision

- Uncontrolled snapshot
- Values are mixture of different (unknown factors)
- Many occlusion due to perspective
- Scale unknown

Many complications of computer vision arise in different form also in a medical setting.

Structure in images

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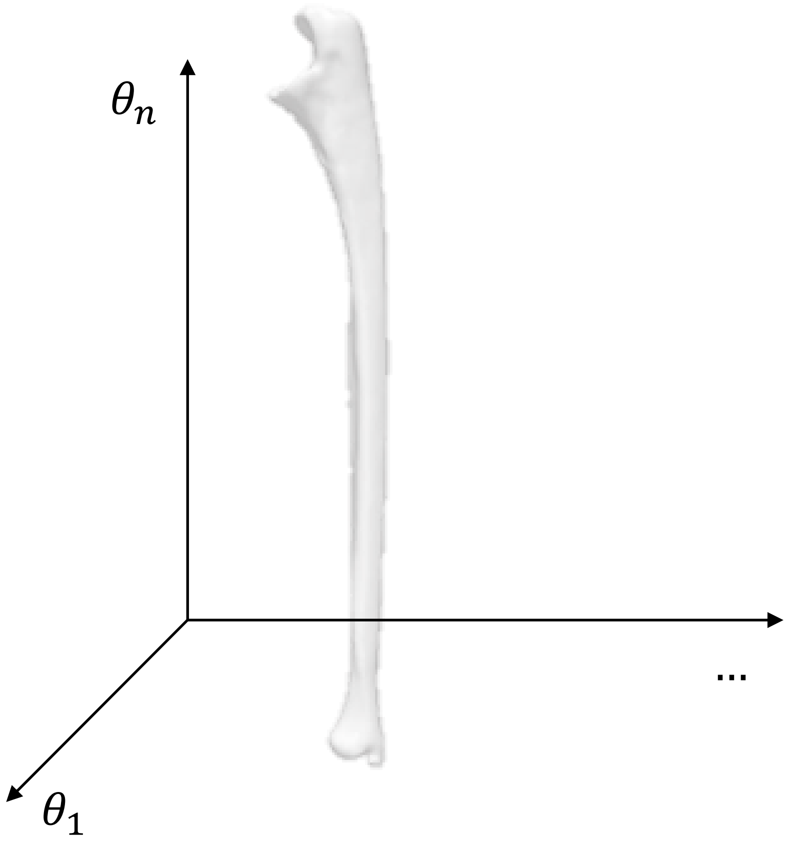
Our mission:
- Model this structure
  - Needs only few parameters
- Explain image by finding appropriate parameters that reflect objects / laws / processes

Analysis by Synthesis in 5 (simple) steps

Analysis by synthesis in 5 simple steps

1. Define a parametric model

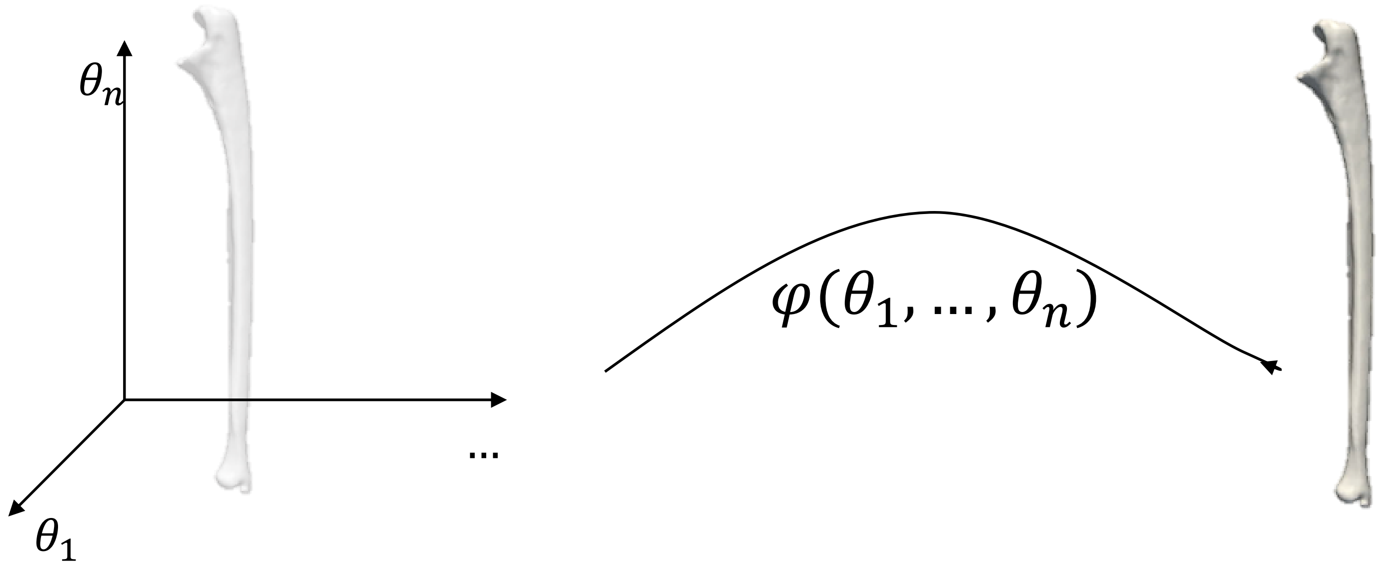
- a representation of the world
- State of the world is determined by parameters $\theta = (\theta_1, \dots, \theta_n)$



Analysis by synthesis in 5 simple steps

2. Define a synthesis function $\varphi(\theta_1, \dots, \theta_n)$

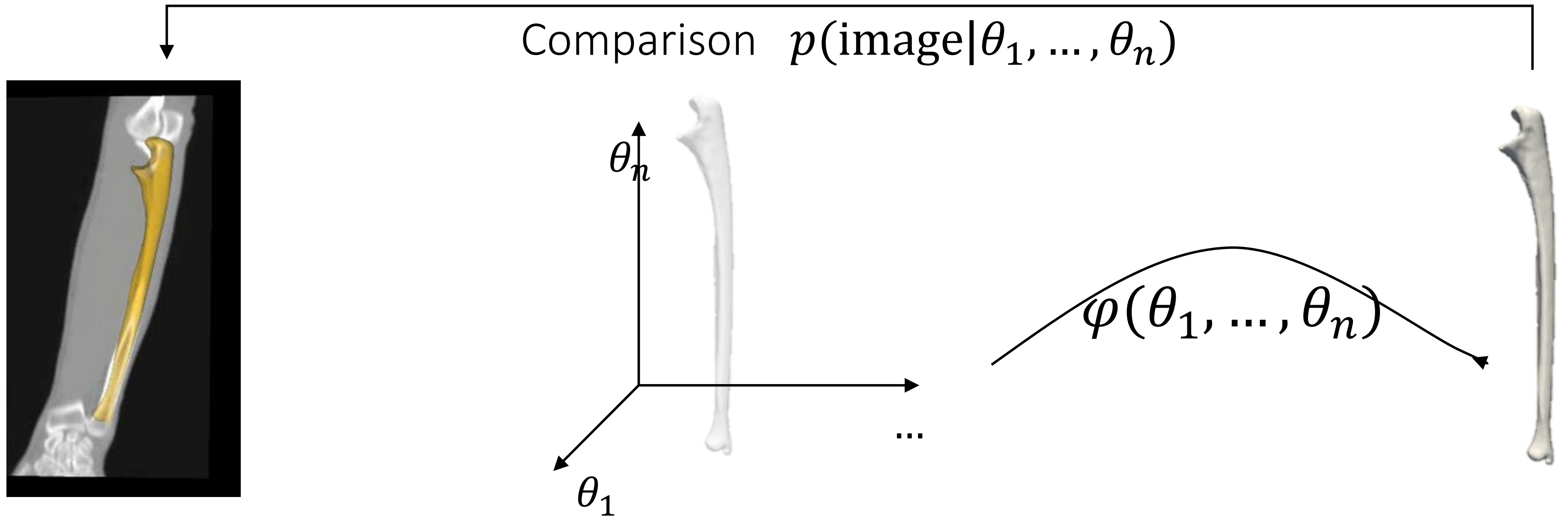
- generates/synthesize the data given the "state of the world"
- φ can be deterministic or stochastic



Analysis by synthesis in 5 simple steps

3. Define likelihood function:

- Define a probabilistic model $p(\text{image}|\theta_1, \ldots, \theta_n)$ that models how the synthesized data compares to the real data
- Includes stochastic factors on the data, such as noise



Bayesian inference

We have: $P(image|\theta_1, \dots, \theta_n)$
We want: $P(\theta_1, \dots, \theta_n|image)$

Bayes rule:

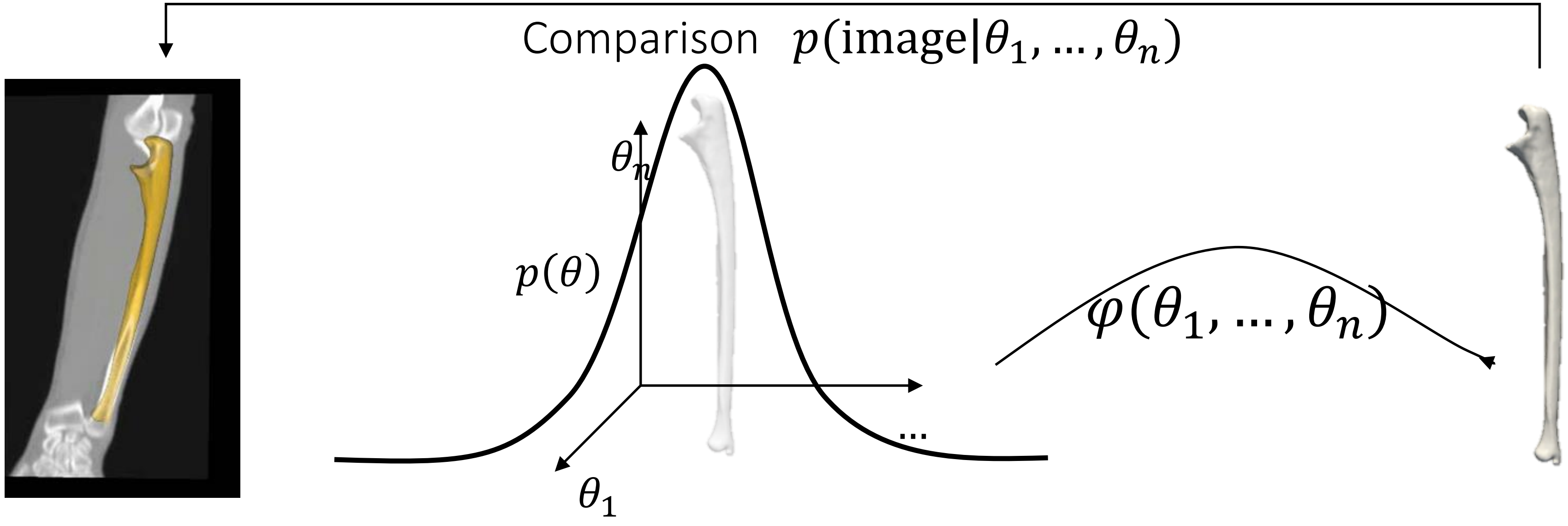
$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)}$$

Lets us compute from $p(D|\theta)$ *its "inverse"* $p(\theta|D)$

Analysis by synthesis in 5 simple steps

4. Define prior distribution: $p(\theta) = p(\theta_1, \dots, \theta_n)$
    - Our believe about the "state of the world"

- Makes it possible to invert mapping $p(\text{image}|\theta_1, \dots, \theta_n)$



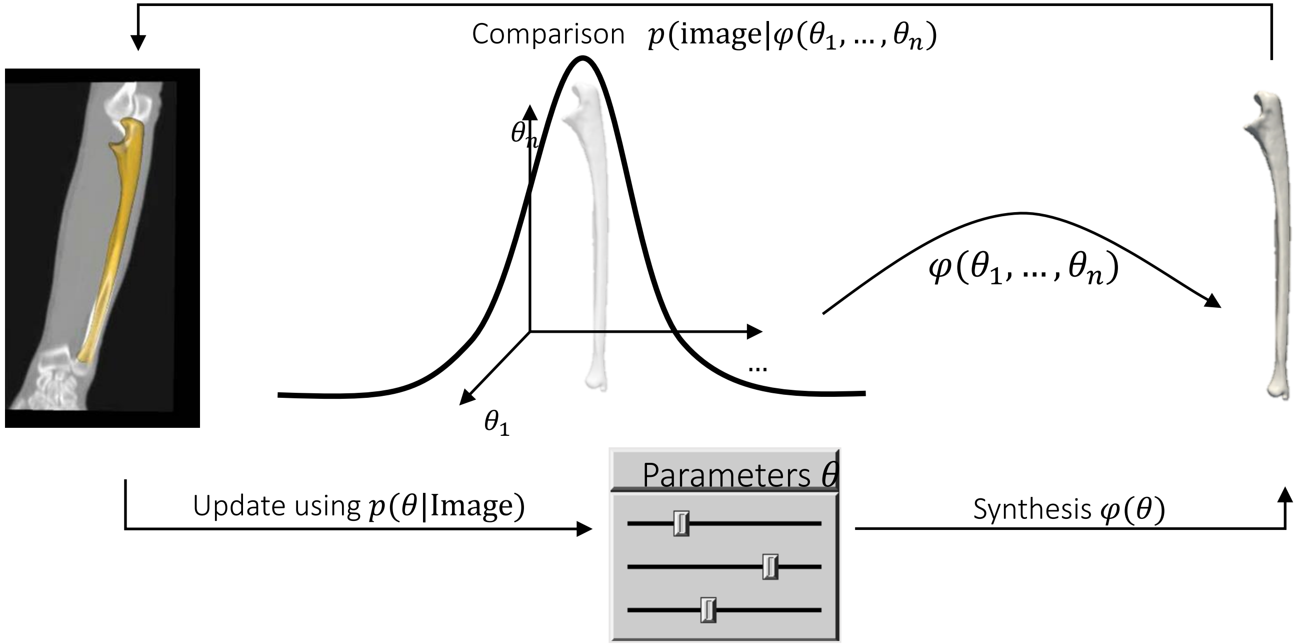
Analysis by synthesis in 5 simple steps

5. Do inference

$$p(\theta_1, \dots, \theta_n | \text{image}) = \frac{p(\theta_1, \dots, \theta_n) p(\text{image} | \theta_1, \dots, \theta_n)}{p(\text{image})}$$

Purely conceptual formulation:

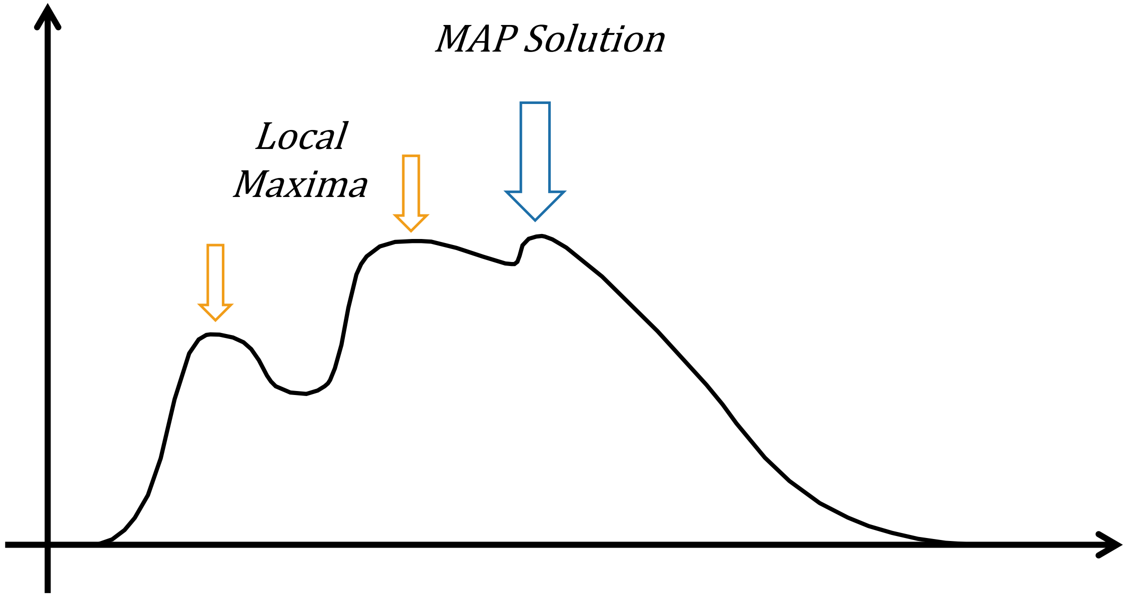
- Independent of algorithmic implementation
- But usually done iteratively



Analysis by synthesis in 5 simple steps

5. Possibility 1: Find best (most likely) solution:

$$\underset{\theta_1,\ldots,\theta_n}{\arg\max}\, p(\theta_1, \ldots, \theta_n | \text{image}) = \underset{\theta_1,\ldots,\theta_n}{\arg\max} \frac{p(\theta_1, \ldots, \theta_n) p(\text{image} | \theta_1, \ldots, \theta_n)}{p(\text{image})}$$



Most popular approach

- Usually based on gradient-descent
- May miss good solutions

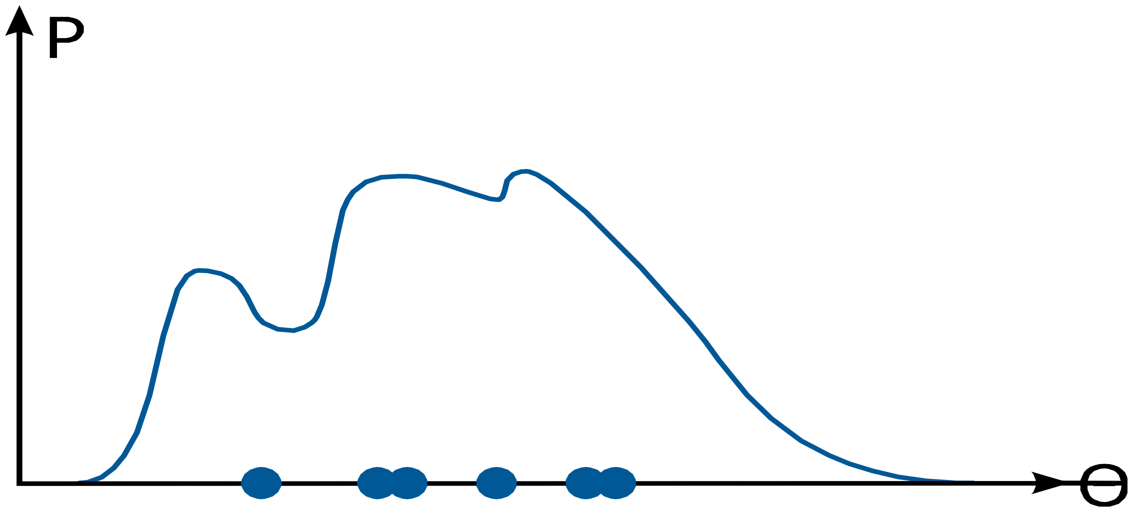
Analysis by synthesis in 5 simple steps

5. Possibility 2: Find posterior distribution:

$$p(\theta_1, \dots, \theta_n | \text{image}) = \frac{p(\theta_1, \dots, \theta_n) p(\text{image} | \theta_1, \dots, \theta_n)}{p(\text{image})}$$

Core of this course

- Obtain samples from the distribution
- Based on Markov Chain Monte Carlo methods



First example: Image registration