Curvature Guided Level Set Registration using Adaptive Finite Elements

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Abstract. We consider the problem of non-rigid, point-to-point registration of two 3D surfaces. To avoid restrictions on the topology, we represent the surfaces as a level-set of their signed distance function. Correspondence is established by finding a displacement field that minimizes the sum of squared difference between the function values as well as their mean curvature. We use a variational formulation of the problem, which leads to a non-linear elliptic partial differential equation for the displacement field. The main contribution of this paper is the application of an adaptive finite element discretization for solving this non-linear PDE. Our code uses the software library DUNE, which in combination with pre- and post-processing through ITK leads to a powerful tool for solving this type of problem. This is confirmed by our experiments on various synthetic and medical examples. We show in this work that our numerical scheme yields accurate results using only a moderate number of elements even for complex problems.

1 Introduction

Virtually all methods in pattern recognition and image analysis rely on prior knowledge about the problem to be solved. Often, this prior knowledge is given in the form of statistical information acquired from a set of representative examples. In order to be able to extract meaningful information from several objects of a class, the objects have to be brought into correspondence. That is, to every point in a reference object, one needs to find the corresponding point in all the examples. The problem of establishing correspondence is known as the registration problem.

In this article we consider the problem of dense point-to-point registration of two 3D surfaces. Surface registration is a common problem and has been researched extensively (see [6] for a comprehensive survey). Most common approaches to surface registration are either formulated directly in terms of the given surface triangulation or require the surfaces to be parameterized. The approach we propose in this paper is to represent the surfaces as the zero-level set of the signed distance function to the surface. This formulation yields a problem description that is independent of the topology of the surface. Further, it leads naturally to a variational formulation and allows us to apply the powerful mathematical methods developed in this field (see e.g. [9]). While our method is general and can be applied to many surface registration tasks, our particular motivation stems from two problems in medical imaging. The goal is to build a statistical model of the human skull and the femur bone respectively. The human skull is a complex structure and finding a suitable surface parametrization is deemed infeasible. For registration of the femur, the advantage of our representation is that correspondence is established for a neighborhood around the surface, which helps later to fit the inner structures of the bones.

The idea of surface registration using a level-sets representation of the surfaces has been described earlier [18, 14]. For the mathematical formulation, our contribution is the inclusion of an additional curvature term in the model that drives the registration in direction tangential to the surface, similar to [12]. The difference to our work is that the curvature is calculated on the parametrization, while we extend the curvature feature to the whole space.

This formulation and its relation to the well known Thirion's Demons algorithm [17] has been detailed in [13]. The main contribution of this paper is a memory-efficient and flexible representation of the data using adaptive finite elements together with its numerical implementation using the DUNE library [2]. The finite element representation gives the flexibility to represent fine details where this is needed (e.g. around the surface) while providing a sparse representation of the function. Further, the numerical method can be easily parallelized.

This paper is structured as follows: In Section 2 we present the mathematical model of our approach. Section 3 describes the finite element discretization and the numerical procedure we employ to solve the registration problem. The feasibility of our approach is illustrated in Section 4 where we show registration results for medical 2D and 3D examples. A more detailed study of the algorithm including variation of parameters and a comparison with a finite difference implementation in ITK [11] is published in [3].

2 Mathematical Model

In this section we present the mathematical model we use for surface registration. In general, registration is an ill-defined problem. The notion of correspondences can greatly vary for different applications. For our application, we define three criteria a good registration has to fulfill: 1) the surfaces should be accurately matched, 2) the curvature at corresponding points should be similar and 3) the deformation should be smooth. In the remainder of this section, we will make these notions precise.

2.1 Level-Set Representation

A common way to model a surface is by representing it as the zero level set of an auxiliary function $I : \mathbb{R}^n \to \mathbb{R}$. This means that the surface Γ is given as:

$$\Gamma := \{ x \in \mathbb{R}^n \mid I(x) = 0 \}.$$

 $\mathbf{2}$

The main advantage of the level-set representation is the independence of the surface's topology. In practice, the most common choice for representing a given surface $\Gamma \subset \mathbb{R}^n$ through a level set function is to use the signed distance function to Γ :

$$I(x) := d_{\Gamma}(x) = \begin{cases} \operatorname{dist}(x, \Gamma) & x \in \operatorname{outside}(\Gamma) \\ 0 & x \in \Gamma \\ -\operatorname{dist}(x, \Gamma) & x \in \operatorname{inside}(\Gamma), \end{cases}$$
(1)

where $\operatorname{dist}(x, \Gamma)$ is the Euclidean distance from x to Γ and the inside and outside of Γ have to be assigned in some meaningful way. When calculated on a rectangular domain $\Omega \subset \mathbb{R}^n$, the distance function can be interpreted as an image over Ω . This leads to the problem of intensity based, non-rigid image registration. In fact our formulation of the problem has been derived from Thirion's Demon algorithm, one of the most widely used image-registration algorithms.

2.2 Thirion's Demons

In his landmark paper, Thirion [17] proposed a method for three-dimensional, non-rigid image registration. Originally formulated in a heuristic manner as an optical flow like algorithm, it was later rigorously studied and formalized. In particular, Modersitzki [15] as well as Cachier et al. [16], have presented variational formulations of the Demons Algorithm, on which we base our work.

The Demons algorithm corresponds essentially to the variational problem of minimizing the functional

$$\mathcal{J}[u] = \mathcal{D}[u] + \alpha \mathcal{R}[u]$$

where

$$\mathcal{D}[u] = \frac{1}{2} \int_{\Omega} \frac{1}{Q_I(x)} \left(I_0(x + u(x)) - I_1(x) \right)^2 dx$$

is a distance measure, and

$$\mathcal{R}[u] = \frac{1}{2} \sum_{l=1}^{3} \int_{\Omega} |\nabla u_l|^2 \, dx$$

is a regularization term. Here I_0 and I_1 are the images defined on Ω and $u: \Omega \to \mathbb{R}^3$ is the displacement field to be calculated. The parameter $\alpha \in \mathbb{R}$ controls the influence of the regularizer. The weight Q_I is chosen as $Q_I(x) = |\nabla I_0(x)|^2 + (I_0(x) - I_1(x))^2$, motivated by Thirion's original formulation. See [16] for a detailed discussion and interpretation of this term.

The registration problem is thus to find the deformation field u, that solves the following variational problem:

$$\mathcal{J}[u] = \mathcal{D}[u] + \alpha \,\mathcal{R}[u] \to \min.$$
⁽²⁾

From the calculus of variations, it is known that any solution has to fulfill the Euler Lagrange equation:

$$\frac{1}{Q_I(x)} \left(I_0(x+u(x)) - I_1(x) \right) \nabla I_0(x+u(x)) - \alpha \Delta u(x) = 0, \ \forall x \in \Omega.$$
(3)



Fig. 1: Two skulls colored according to their mean curvature.

This is a non-linear elliptic partial differential equation, which can, for example, be solved using the numerical method presented in Section 3.

2.3 Curvature guided registration

Thirion's Demons algorithm was designed for the registration of medical images (e.g. CT images), that feature meaningful information on the whole domain. In our approach the only information comes from the surface that represents the zero-level set. Furthermore, on the zero-level set, the value is by definition zero everywhere. We have no information about features that could guide the registration in surface direction. Hence corresponding points are, apart from the influence of the smoothing term \mathcal{R} , only sought in the direction normal to the level sets. The resulting correspondences on the zero-level do therefore not necessarily correspond to meaningful features. For a large class of objects, corresponding points in two surfaces have similar curvature. Therefore, we use the mean curvature at a point as an additional feature to be matched. Figure 1 illustrates that for registration of human skulls, the curvature is indeed a reasonable feature to include.

We extend the functional including an additional term which leads to a matching of the curvature

$$\mathcal{C}[u] := \frac{1}{2} \int_{\Omega} \frac{1}{Q_H(x)} (H_0(x+u(x)) - H_1(x))^2 \, dx.$$

where $H_0(x)$ and $H_1(x)$ are the mean curvatures at point x for I_0 and I_1 , respectively. The weight $Q_H(x)$ is chosen analogously to $Q_I(x)$. The registration problem is now to find u that solves the following problem:

$$\mathcal{J}[u] := \mathcal{D}[u] + \beta \mathcal{C}[u] + \alpha \mathcal{S}[u] \to \min.$$
(4)

The Euler-Lagrange equation is extended in the obvious way, leading to

$$-\alpha \triangle u = F(u) \tag{5}$$

with

$$F(u) := \frac{I_0(x+u(x)) - I_1(x)}{Q_I(x)} \nabla I_0(x+u(x)) + \frac{H_0(x+u(x)) - H_1(x)}{Q_H(x)} \nabla H_0(x+u(x)) ,$$

3 Finite Element Discretization

In this section we describe the steps taken for computing the solution u for given data I_0, I_1, H_0 , and H_1 from (5). In [15], Modersitzki showed that the Demons algorithm can be interpreted as a simple fix point iteration scheme for the nonlinear elliptic equation (5). The solution u is obtained from an initial solution u^0 by iteratively performing the computation

$$u^{n+1} = u^n + \tau \left(\alpha \triangle u^n + F(u^n) \right).$$

Alternatively, we can interpret this equation as a forward Euler step for the heat equation

$$\partial_t u - \alpha \triangle u = F(u) \tag{6}$$

with step size τ . In this presentation we will focus on deriving methods for computing large time solutions u = u(t, x) of (6). Since we are interested in the large time limit we use as the initial conditions simply u(0, x) = 0 in all our calculations. For the simulations shown here we have used Neumann boundary conditions.

Since the elliptic operator in the heat equation leads to a severe time step restriction, coupling the time step τ to the mesh width h via $\tau = O(h^2)$, we use an implicit time discretization for the elliptic part of (6). To avoid problems with the nonlinear term F(u) we want to discretize this term in an explicit fashion. Fixing a time step τ , and using the abbreviation $u^n(x) \approx u(n\tau, x)$ we propose the following semi-implicit scheme:

$$u^{n+1} - \tau \alpha \triangle u^{n+1} = u^n + \tau F(u^n). \tag{7}$$

This approach is similar to Thirion's approach with the exception that the elliptic term is treated implicitly. Similarly higher order implicit/explicit Runge-Kutta schemes for the time discretization can be used [5].

It remains to specify the spatial finite element discretization of the image domain. We use a Discontinuous-Galerkin Finite-Element approach. This method is very well suited for this type of problem and can be easily used with locally adapted grids and domain decomposition strategies for parallelization on distributed memory computers. Given a tessellation $\mathcal{T}_h = \{T_i\}_{i \in \mathcal{I}}$ of the computational domain Ω into non overlapping elements (see Figures 4 and 2a in the following Section), this scheme follows the same ideas as the standard Galerkin method [7] but employs a discontinuous ansatz space: $V_h^k := \{v_h : v_i \in$ $P_k(T_i)$ for $i \in \mathcal{I}$ }. Here $v_i \equiv v_{h|\mathcal{T}_h}$ and $P_k(T_i)$ denotes the space of polynomials on the element T_i of order k. Note that there are no continuity assumptions between elements.

Now, a variational formulation for the implicit and the explicit part

$$L_{\text{impl}} := u - \tau \alpha \triangle u, \quad L_{\text{expl}} := u + \tau F(u)$$
(8)

of the semi-implicit scheme (7) is derived: The explicit part is easily discretized on an element T_i by:

$$\int_{T_i} L_{\exp l,i} \varphi = \int_{T_i} (u_i + \tau F(u_i)) \varphi \, dx \tag{9}$$

for all $\varphi \in P_k(T_i)$.

Due to the discontinuous ansatz space, the discretization of the elliptic term is slightly more complicated as in the standard Galerkin approach. We employ the approach known as the *local Discontinuous Galerkin* method, rewriting for given $u_h \in V_h$ the second order equation $L_{impl,h} = u_h - \tau \alpha \Delta u_h$ as a system of first order equations

$$v_h = \nabla u_h, \quad L_{\text{impl},h} = u_h - \alpha \tau \nabla \cdot v_h$$
.

 $L_{\text{impl},h}$ is now computed from the variation formulation:

$$\int_{T_i} v_i \varphi = \int_{\partial T_i} [u_h] \varphi - \int_{T_i} u_h \nabla \varphi , \qquad (10)$$

$$\int_{T_i} L_{\text{impl},i}\varphi = \int_{T_i} u_h \varphi - \int_{\partial T_i} \tau \alpha[v_h]\varphi + \int_{T_i} \tau \alpha v_h \nabla \varphi \tag{11}$$

for all $\varphi \in P_k(T_i)$; we have used the abbreviation $[v_h]$ to denote the jump of a discrete function $v_h \in V_h$ over element boundaries. For more details see [10, 4].

For constructing the tessellation we use the ALUGrid library [1] using hexahedral meshes in 3d and triangular meshes in 2d with non-conforming local adaptivity and the possibility of domain decomposition and dynamic load balancing for parallel computations. The whole numerical scheme is implemented using the generic grid concept from the software library DUNE [2] and the discretization methods from the DUNE-FEM package [8]. Since the DUNE library is implemented in C++ the incorporation of the solution algorithm into the ITK framework [11] presents no major problems so that the pre- and post-processing facilities developed here can be directly used.

4 Results

For the results presented here, the shapes have been aligned prior to registration, to remove large translational and rotational parts. In all the computation we used $\tau = 1, \alpha = 1$, and $\beta = 1$. Using larger values of τ can increase the convergence rate of the numerical scheme and due to the implicit treatment of the elliptic operator does not lead to instability of the scheme; the same holds for smaller values of α but in both cases the smoothness of the displacement field u is decreased in an unsatisfactory manner. For the spatial discretization we have used k = 0, 1, 2 i.e., constant, linear, and quadratic polynomials on each element and also higher order time-discretization schemes. Here, we only show results with k = 1 together with a first order semi-implicit time discretization scheme.



Fig. 2: Registration of two 2D-slices through the femur. Figure 2a shows the outline of the shapes and the discretization of the images, while Figure 2b shows the resulting displacement field.

To increase the rate of convergence and to take advantage of the possibilities offered by local grid adaptation, we start the computation using a coarse grid of less than 100 elements and after performing a number of iterations on this coarse grid, refine the grid elements on which

$$\max\{|I_0(x)|, |I_0(x+u^n(x))|, |I_1(x)|\} < R$$

holds for a given value of R. The indicator R is then decreased and the iteration process is repeated. The full details of the algorithm and a study of the influence of the parameters are published in [3].

4.1 Registration of a femur

As a first test, we register two 2D slices of a 3D femur bone. Figure 2a shows the two shapes to be registered and the locally adapted tessellation of the image domain. The shape of the slice is well matched and the resulting correspondences are reasonable as demonstrated in Figure 2b where we also show the resulting displacement field.

In Figure 3 we see the registration results for the 3D femora from our database. The image shows that the registered image matches the shape of the target accurately. The discretization used is illustrated in Figure 4a. We see that the resolution is highest around the surface and hence we can represent fine details where this is necessary.

4.2 Registration of a skull

As a further example, we consider the registration of two skulls. The discretization used is illustrated in Figure 4b. As previously mentioned, one of the main motivation for the level-set representation was to register surfaces of arbitrary topology. In this example the data is noisy and the topology of the skulls differ due to segmentation artifacts and the limited resolution of the original CT-image.



Fig. 3: The reference femur (a) is registered onto a target (b). In (c), the registration result is shown together with the adaption level (blue=2, red=5).



Fig. 4: The discretization for the representation of the skull surfaces and the femur (refinement levels represented by color).

Still the shape is accurately matched as can be seen in Figure 5. Although not the main motivation of this work, an immediate application is atlas-based labeling of a target skull. This is illustrated in Figure 5, where the mandible is labeled in a reference skull and the labelling is transformed to an unlabelled target skull using the calculated deformation field u. Moreover, this provides us with a test to validate the quality of the registration result. It can be seen, that the mandible is correctly identified in the target skull.

5 Discussion

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Our results demonstrate that even on quite coarse grids and for complex registration prolems, the finite element method leads to very good results. Even the challenge posed by the registration of the human skull was met by the algorithm. The advantage of the local grid adaption for this type of problem is evident, since mainly the neighborhood of the surface must be well resolved while outlying regions can be treated with a far lower resolution without reducing the quality of the match. In the calculation for the 3D femura, the resulting



Fig. 5: The labelling of a reference skull (a) is automatically transformed to a target skull (b), (c).

Computation time for the skull registration was 4.5 hours on a AMD Opteron 2.4GHz.

finest grid consisted of less than 400.000 hexahedra, compared to more than 10M points used in our ITK implementation. A similar reduction was achieved for the skull example. Also the finite element formulation seems to be very robust, so that additional strategies like using smooth low resolution images do not seem to be required for the convergence of the scheme. The implicit treatment of the elliptic part also enhances the stability of the method so that a wide range of parameters can be used with this scheme.

The consequent focus on the formulation of the problem as a PDE offers a wide range of further approaches for computing the displacement field, e.g., higher order schemes or pre-conditioning strategies like multigrid approaches. These can lead to a further increase in the efficiency of the scheme. The DUNE package used for our implementation is based on a generic interface both for the grid structure and the numerical scheme, thus allowing for a generic implementation of the solution method including local adaptivity and dynamic load balancing. We can therefore easily apply different numerical schemes to the registration problem, such as continuous Galerkin discretizations, fully implicit time stepping schemes or direct methods for the non-linear elliptic equation, and compare these with the method presented here. We will study the possibilities offered by this concept in future work.

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