A unified approach to shape model fitting and non-rigid registration

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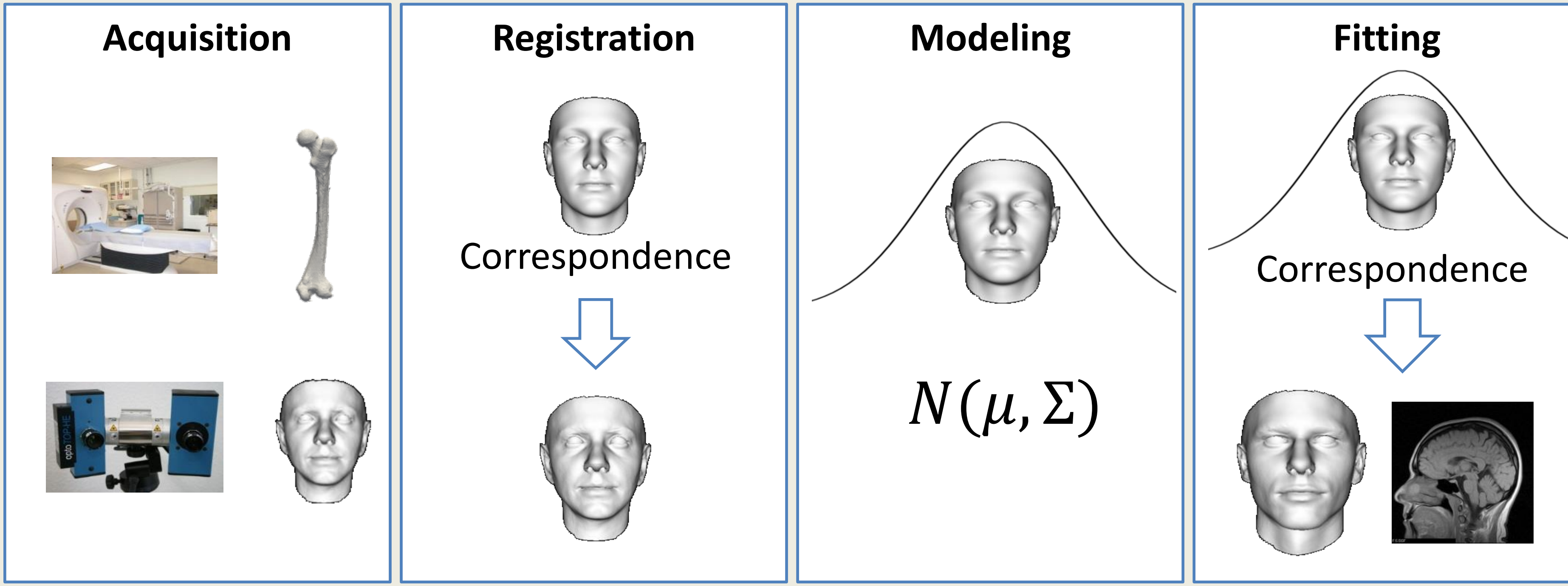
Shape modeling pipeline



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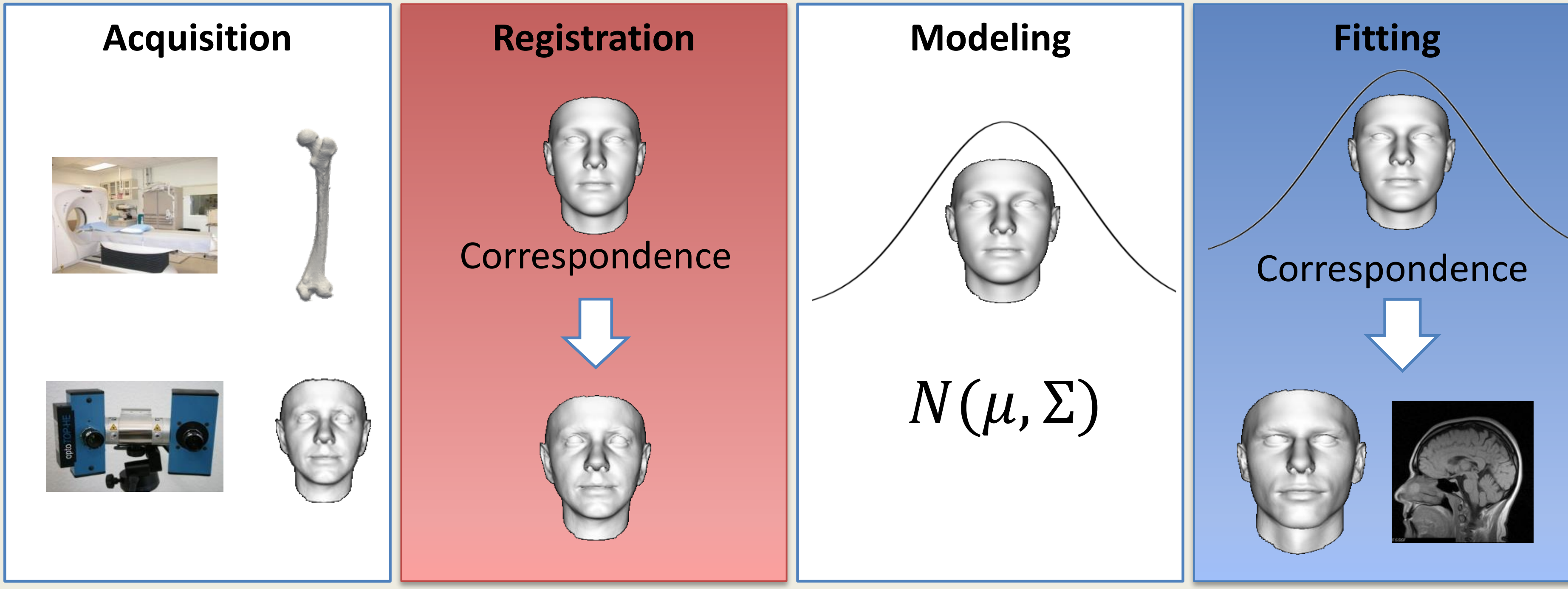
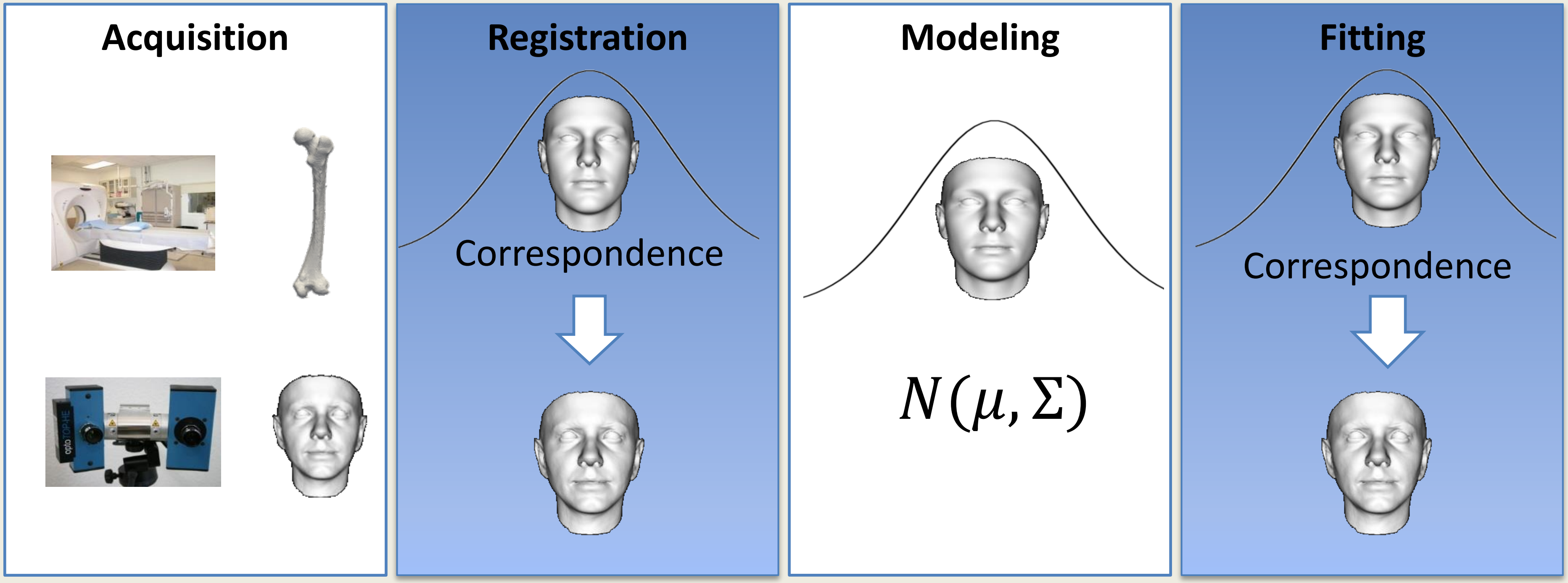


- Weak prior assumptions
- Non-parametric
- Variational approach
- Implicit model (regularization)

- Strong prior
- Parametric
- Standard optimization
- Explicit probabilistic model

Shape modeling pipeline



- Weak prior assumptions
- Parametric
- Standard optimization
- Explicit probabilistic model

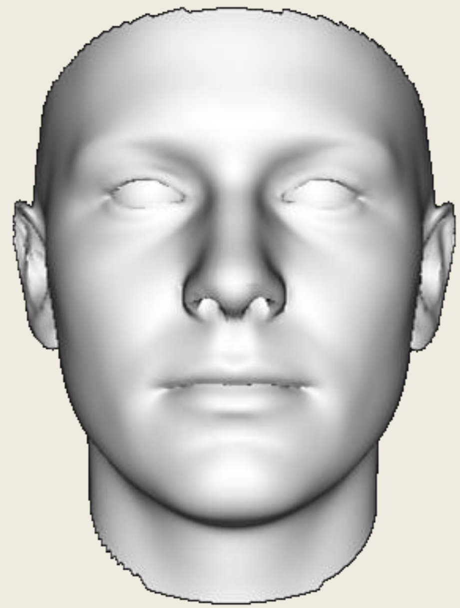
- Strong prior
- Parametric
- Standard optimization
- Explicit probabilistic model

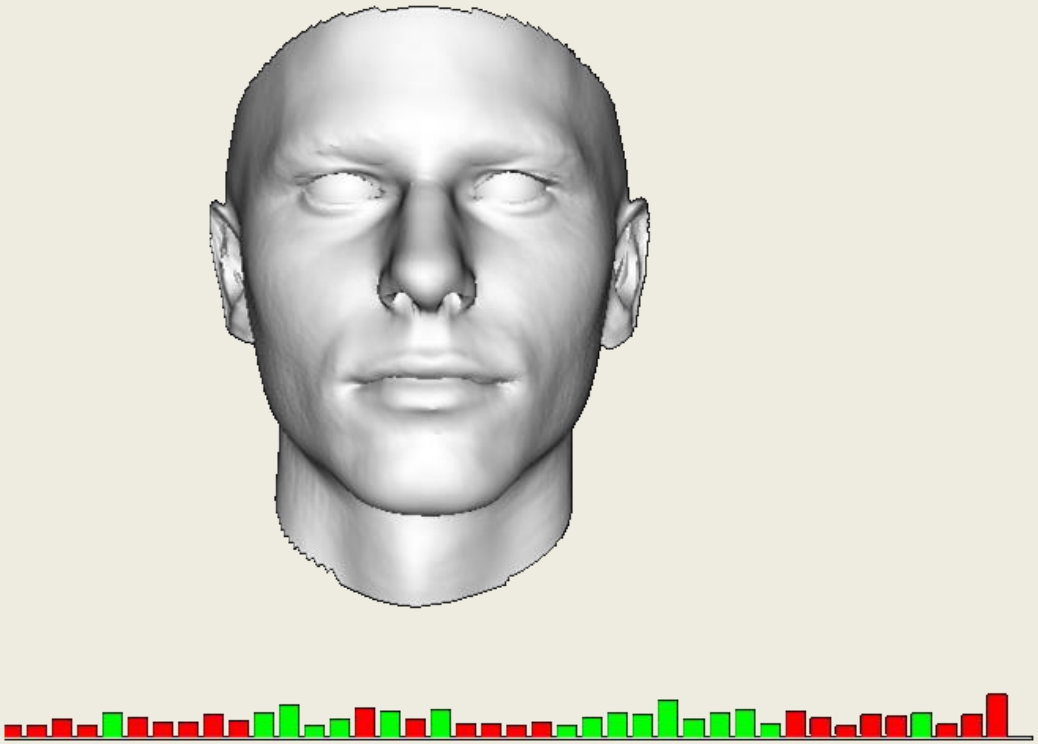
Outline

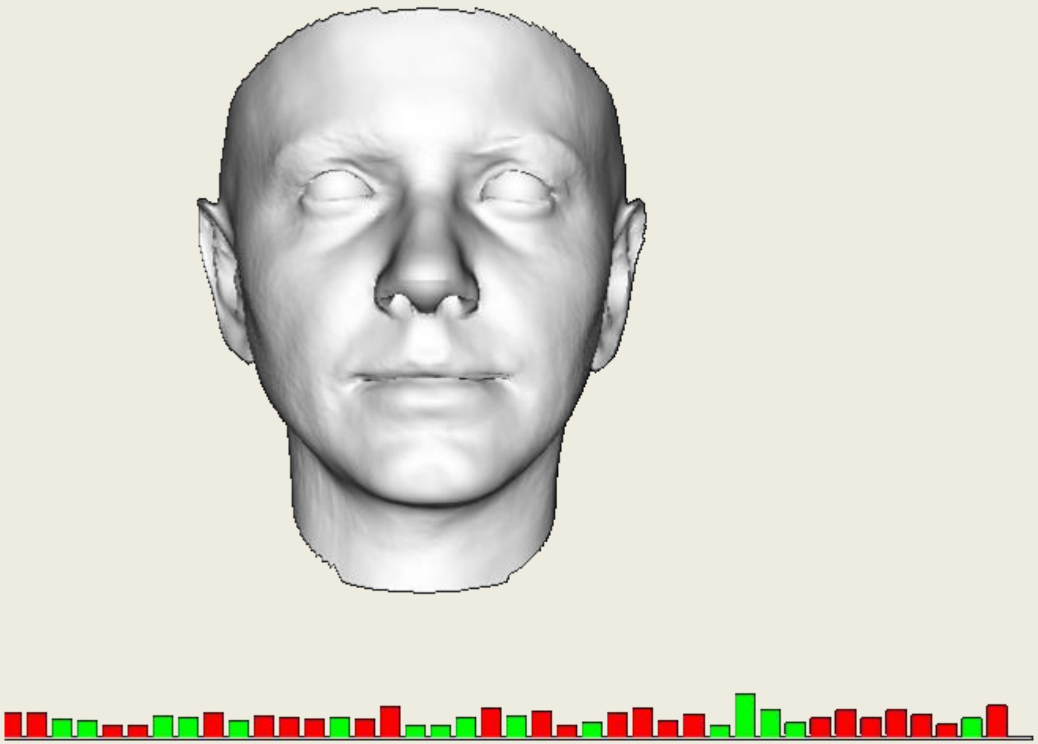
Goal:
Replace registration with model fitting

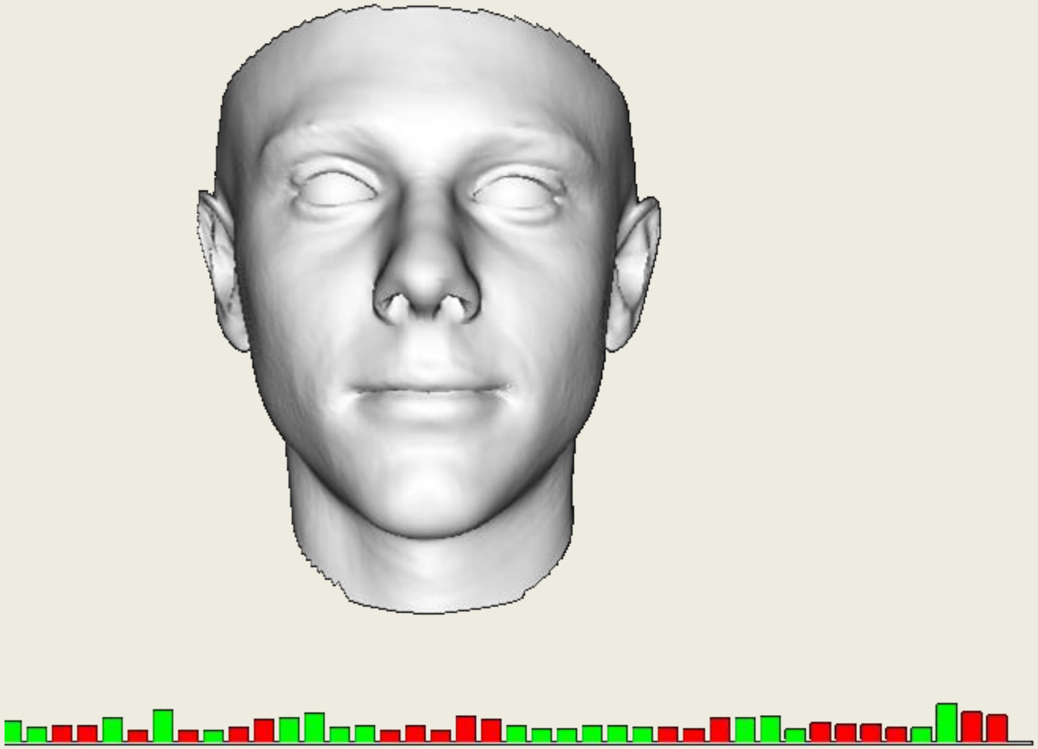
- Why model fitting
- Conceptual formulation
  - Statistical shape models and Gaussian processes
- How to make it practical
  - Low rank approximation
- Application to image registration

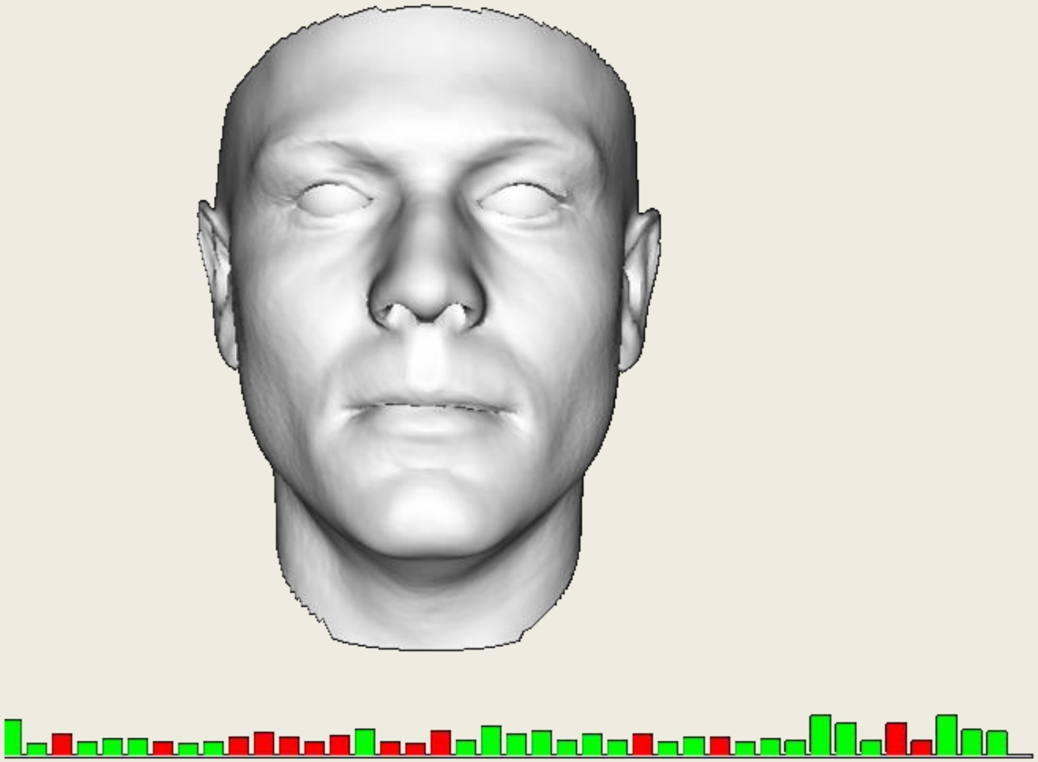
Advantage 1: Sampling

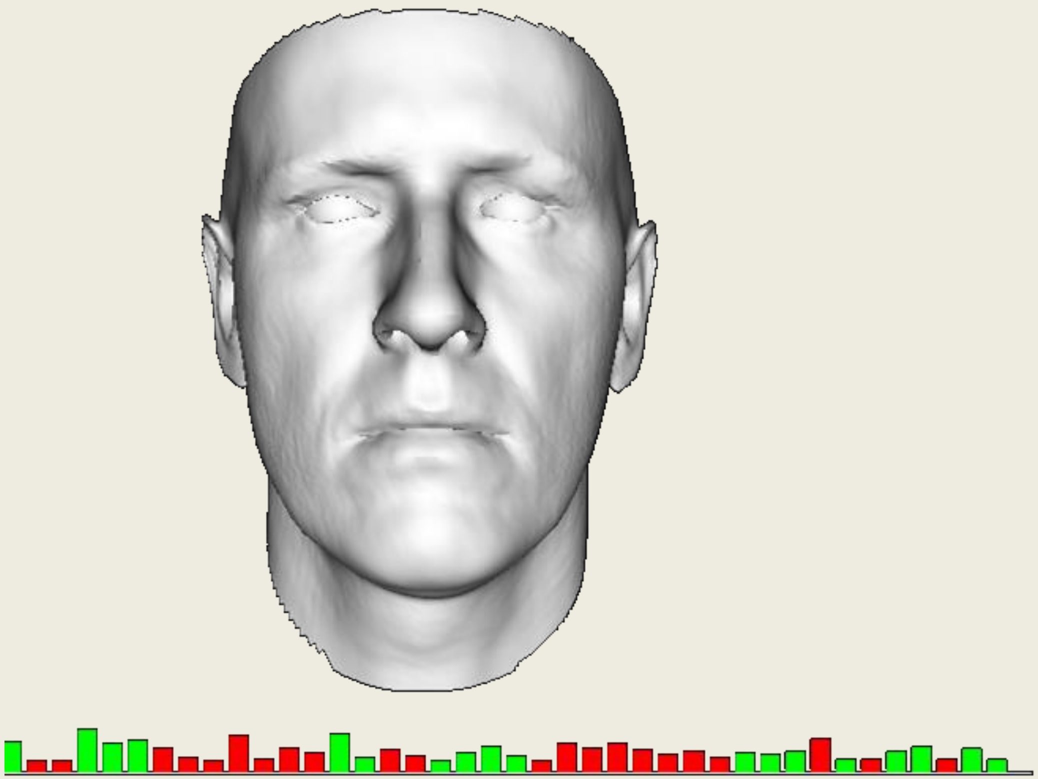




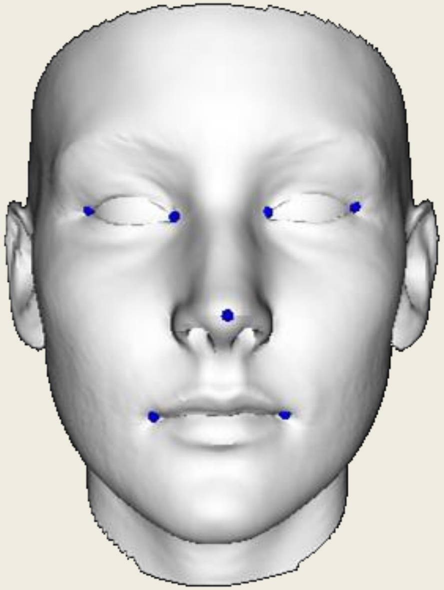


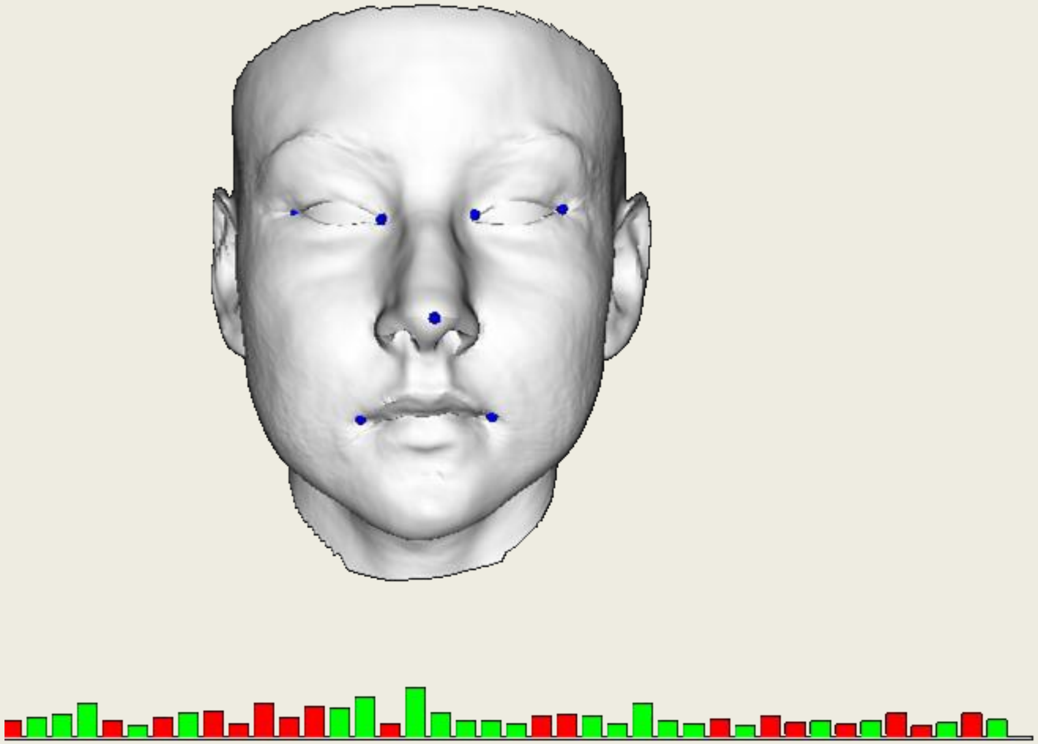


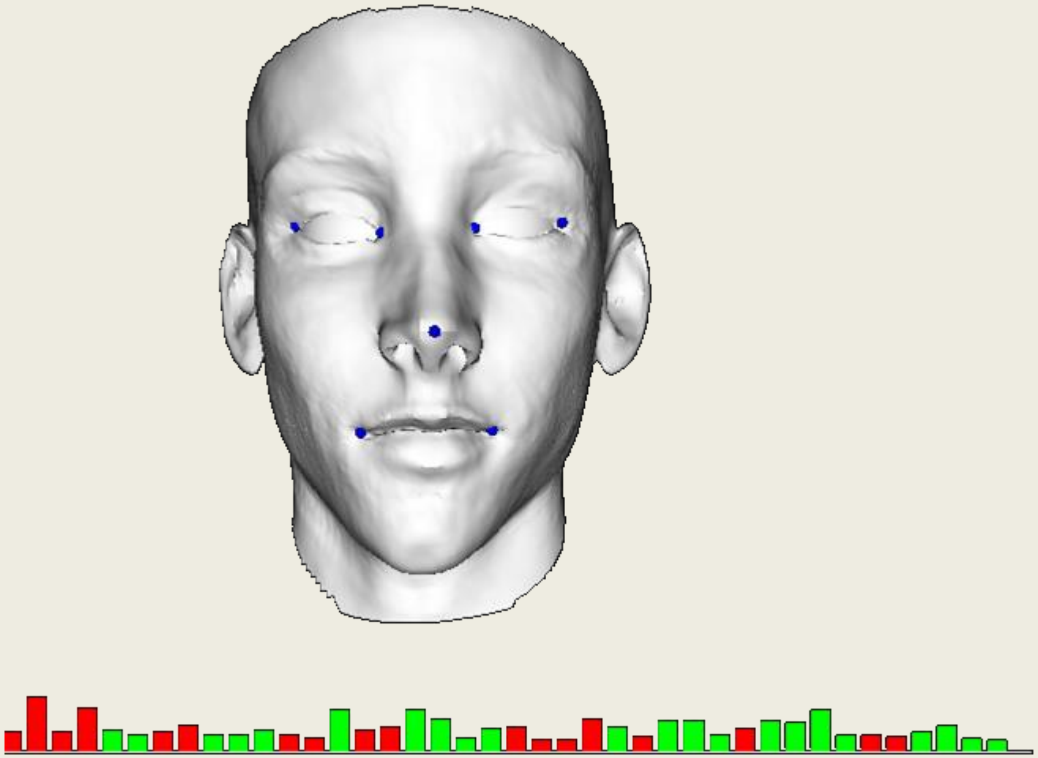


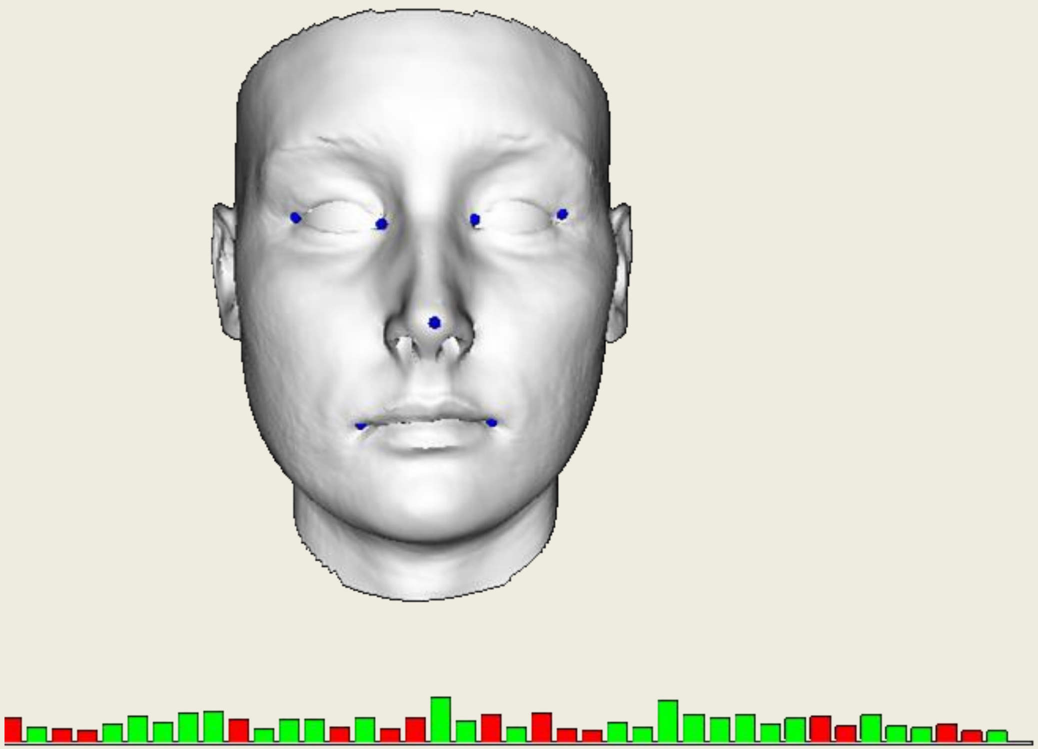


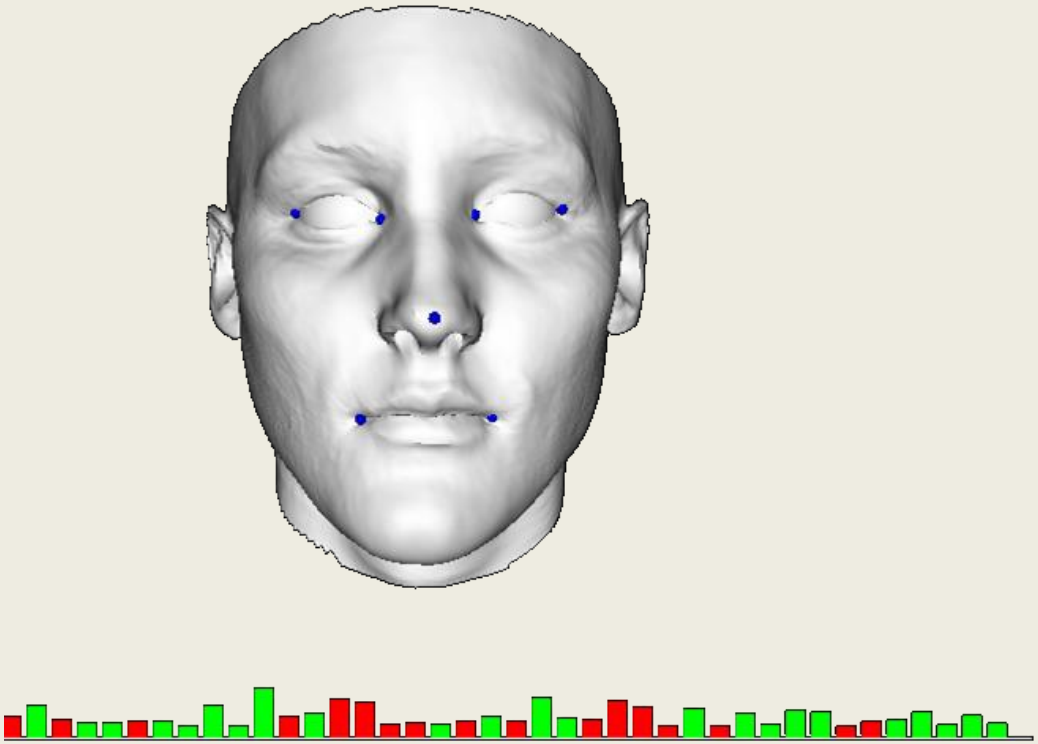
Advantage 2: Posterior models

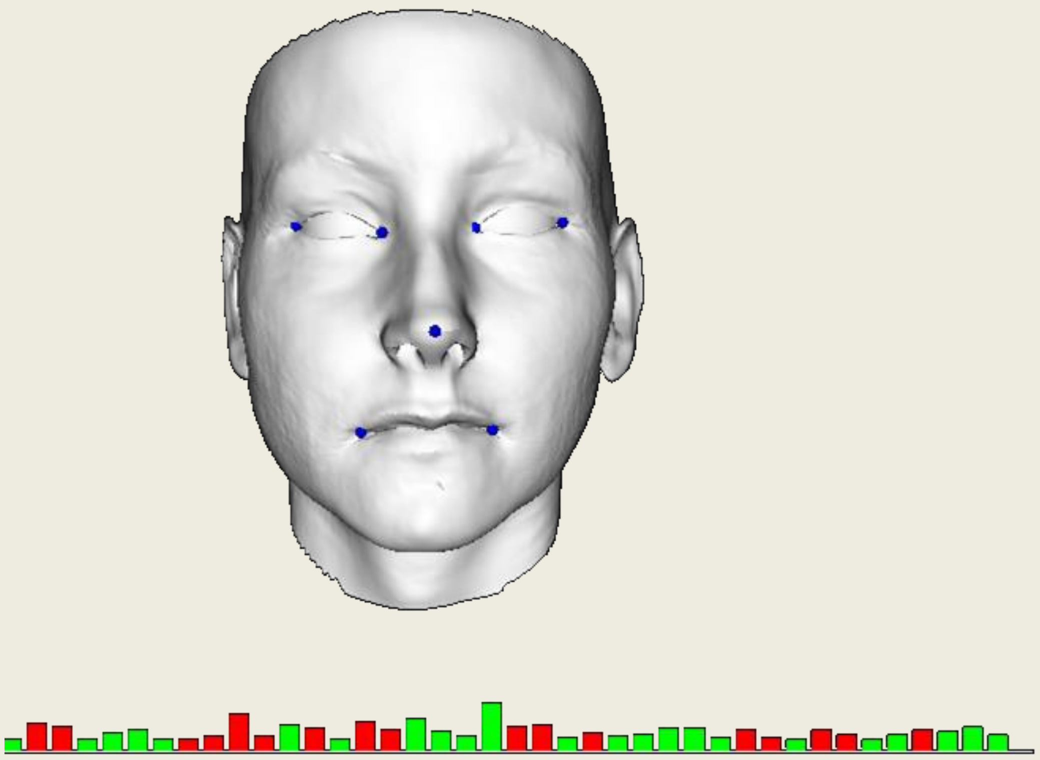




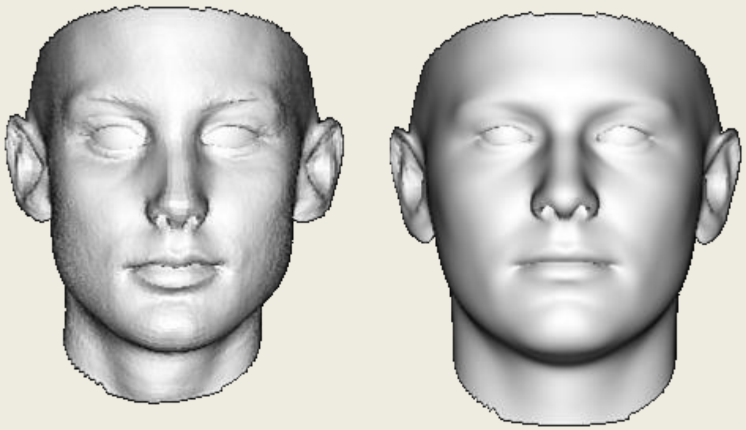


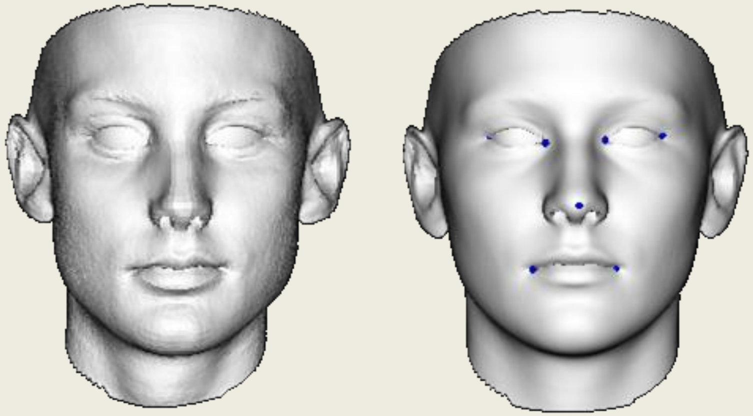


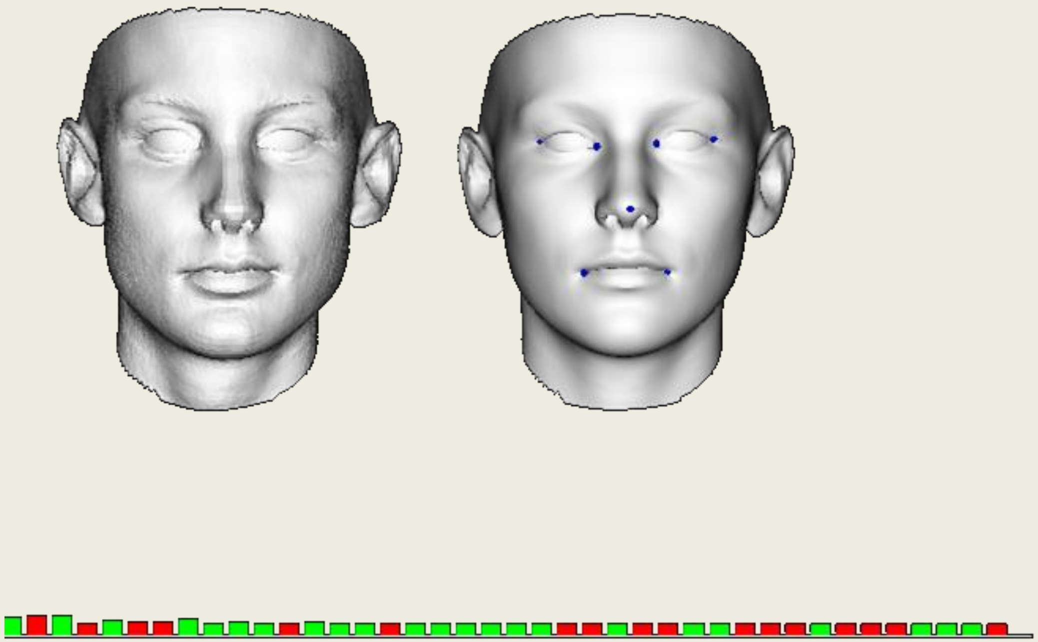


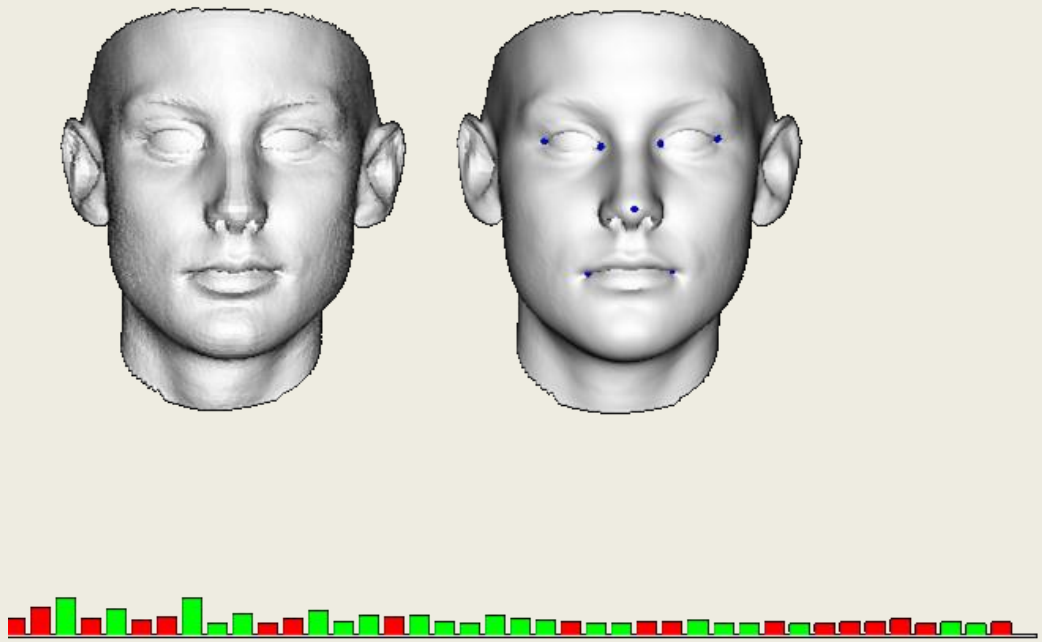


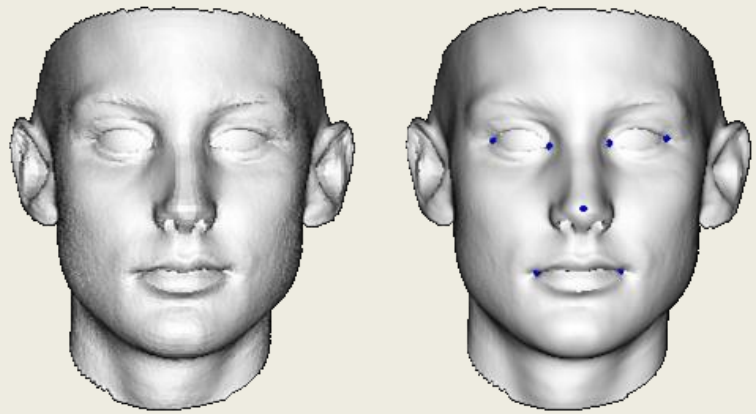
Advantage 3: Simple(r) optimization

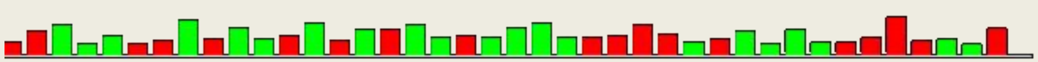


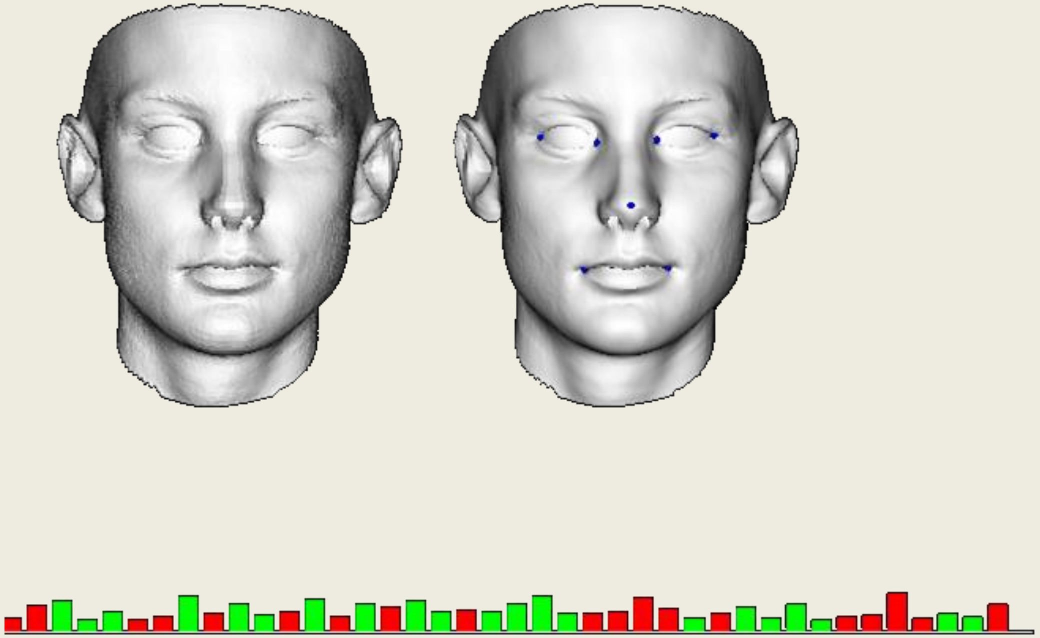


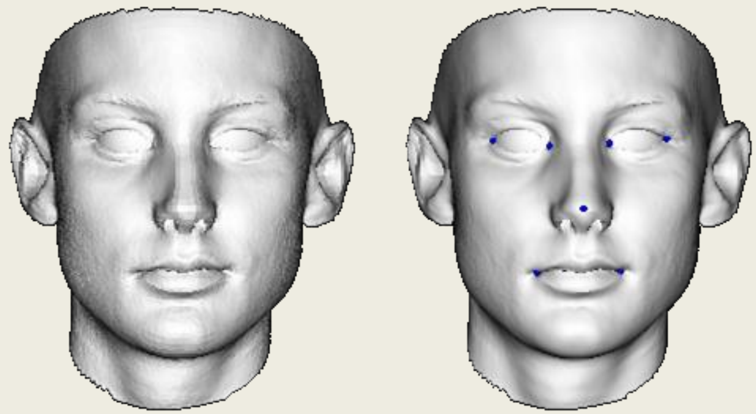


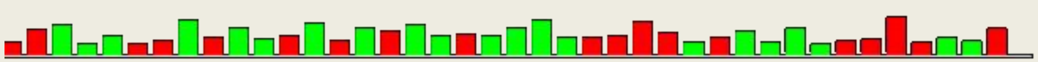












Statistical Shape Models

- Example data:
  Surfaces in correspondence with Reference Γ_R

Γ_1

…

Γ_n

Statistical Shape Models

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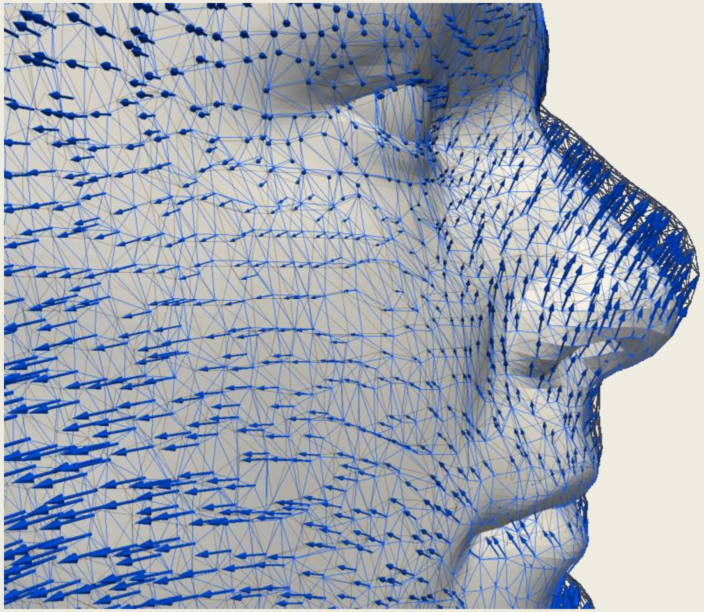
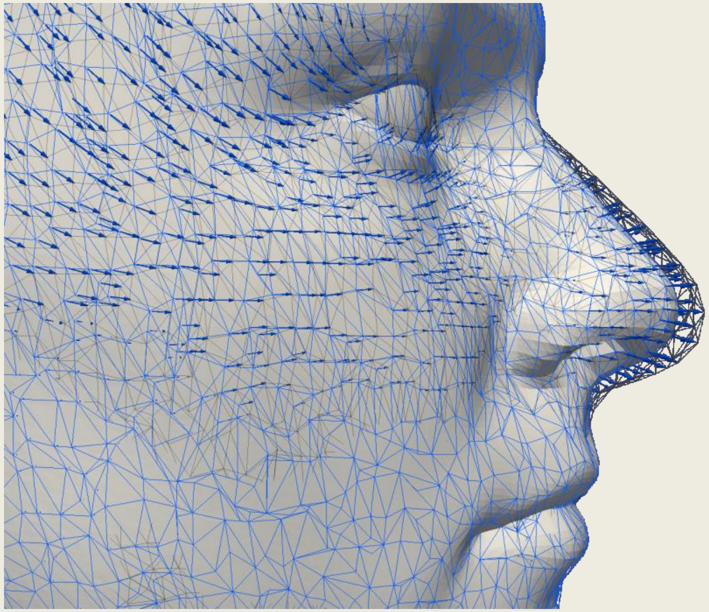
  Surfaces in correspondence with Reference Γ_R





$\Gamma_1 = \Gamma_R + u_1$ … $\Gamma_n = \Gamma_R + u_n$

Statistical Shape Models

- Estimate mean and sample covariance:

$$\mu(x_i) = \frac{1}{n}\sum_k (\overbrace{x_i + u_k(x_i)}^{\Gamma_k(x_i)}) = \overbrace{x_i + \overline{u}_k(x_i)}^{\text{Reference + mean deformation}}$$

$$\Sigma(x_i, x_j) = \frac{1}{n}\sum_k (\overbrace{x_i + u_k(x_i)}^{\Gamma_k(x_i)} - \mu(x_i))(\overbrace{x_j + u_k(x_j)}^{\Gamma_k(x_i)} - \mu(x_j))^T$$

Covariance of deformations

$$= \frac{1}{n}\sum_k (u_k(x_i) - \overline{u}(x_i))(u_k(x_j) - \overline{u}(x_j))^T$$

Gaussian process view

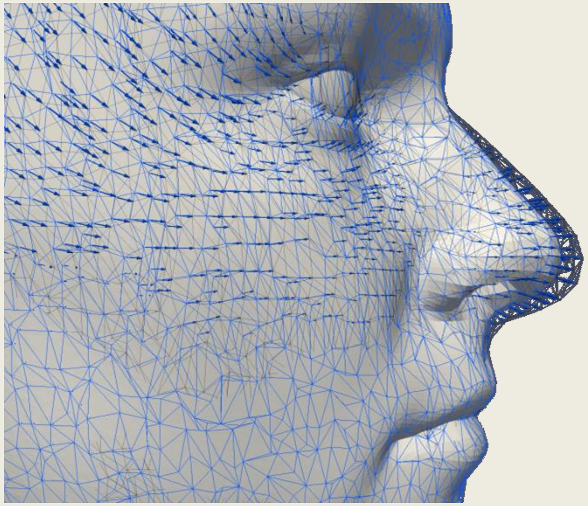
- "Deformation model" on Γ_R

$$\mathrm{u} \sim GP(\overline{u}, \Sigma)$$

$$u: \Gamma_R \to \mathbb{R}^3$$

- Shape model:

$$\Gamma \sim \Gamma_R + u$$



- Model deformations instead of learning them
  - $\Sigma(x, y)$ can be arbitrary p.d. kernel
  - $k(x, y) = \exp(-\frac{\|x - y\|^2}{\sigma^2})$ enforces smoothness

Registration using Gaussian processes

- Previous work:
  - *U. Grenander, and M. I. Miller.*
    Computational anatomy: An emerging discipline.
    Quarterly of applied mathematics, 1998
  - B. Schölkopf, F. Steinke, and V. Blanz.
    Object correspondence as a machine learning problem. *Proceedings of the ICML* 2005.

Challenge:
Space of deformations is very high dimensional

Back to statistical models: PCA

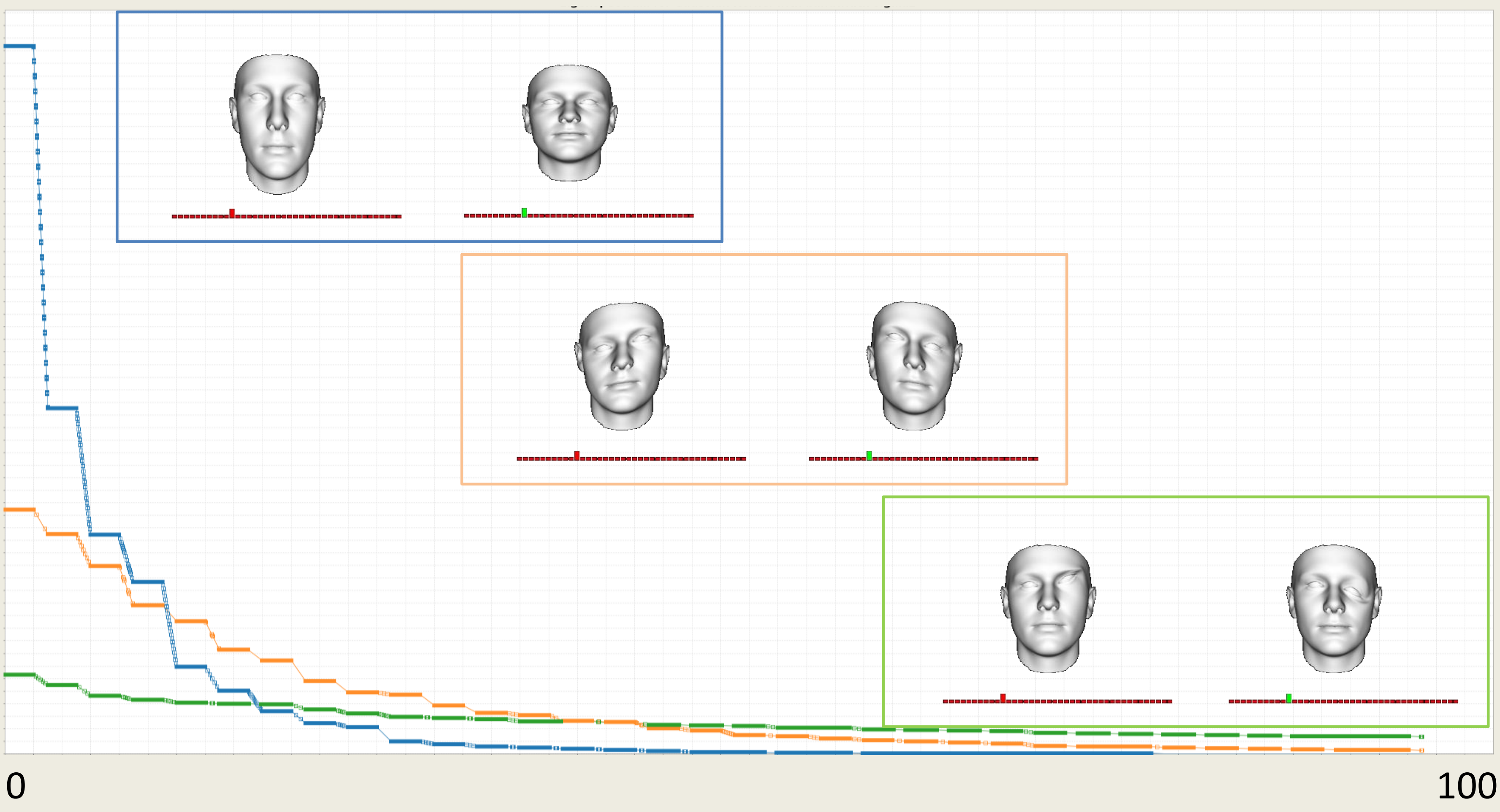
Statistical model $M[\alpha_i, \dots, \alpha_m]$:
$u(x) = \overline{u}(x) + \sum_i^m \alpha_i \sqrt{\tilde{\lambda}_i}\, \tilde{\phi}_i(x), \qquad \alpha_i \sim N(0,1)$

- Mercer's Theorem:

$$k(x,y) = \sum_{i=1}^{n} \lambda_i \phi_i(x) \phi_i(y)$$

- Use Nyström approximation to compute $\left(\tilde{\lambda}_i, \tilde{\phi}_i\right)_{i=1..m}, (\mathrm{m} \ll \mathrm{n})$
- Low rank approximation of k(x,y)

Eigenspectrum and smoothness



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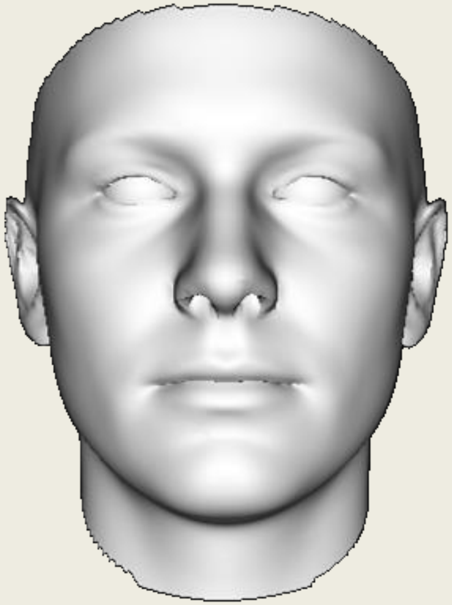
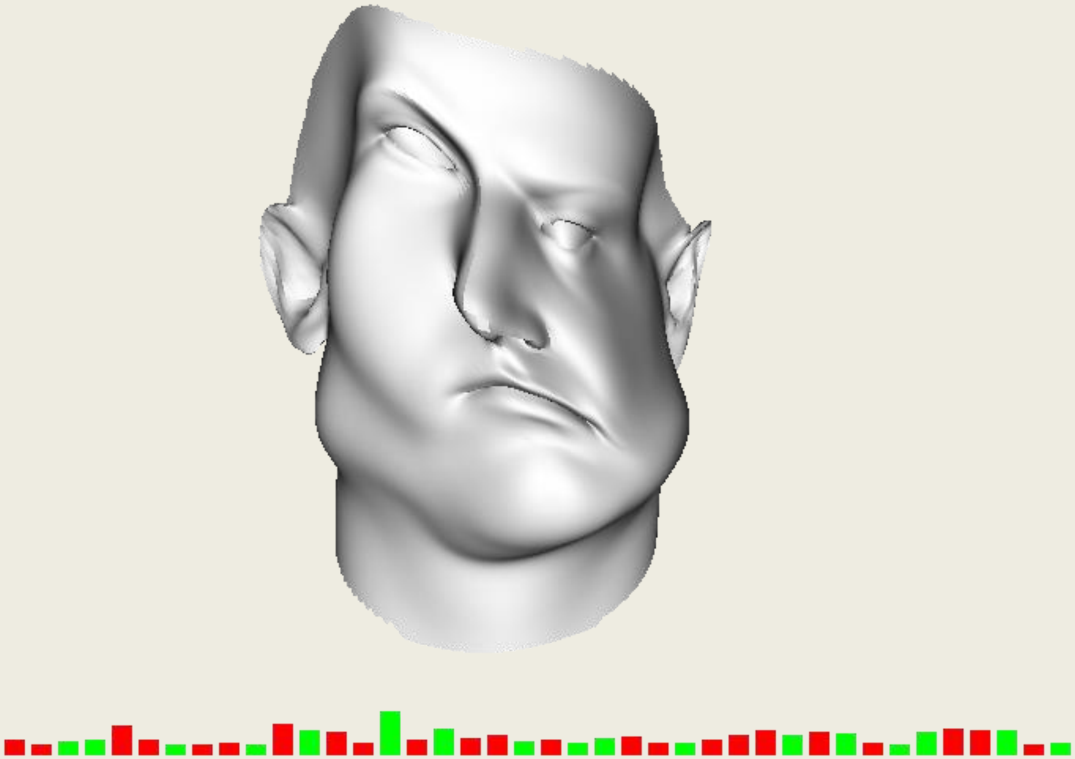
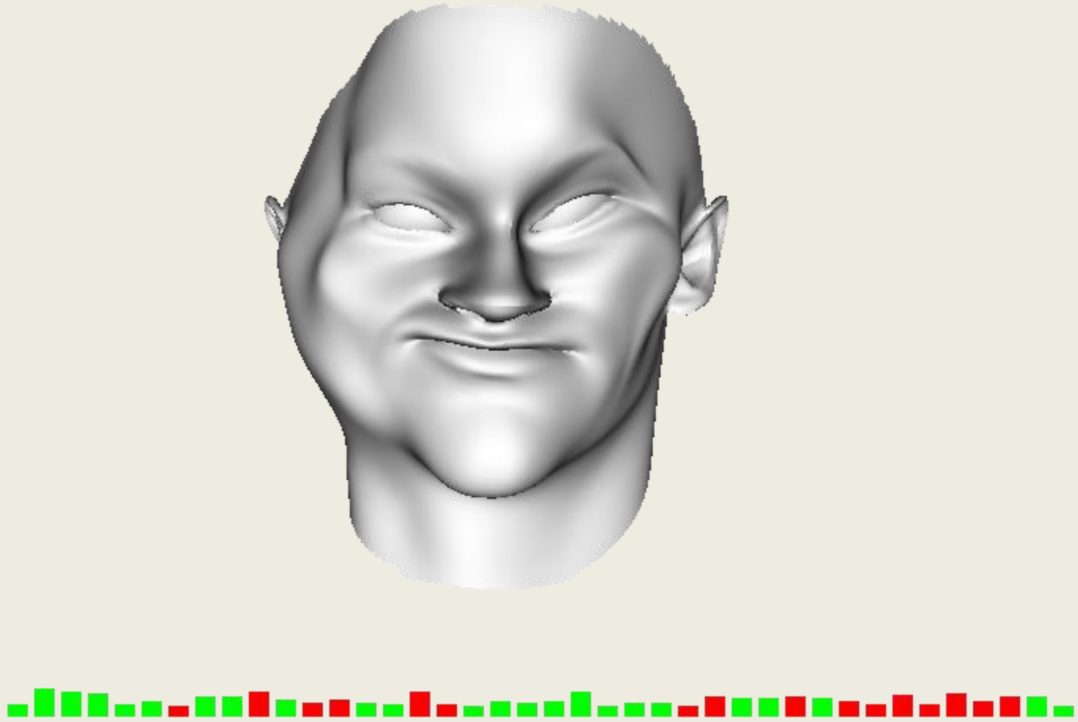
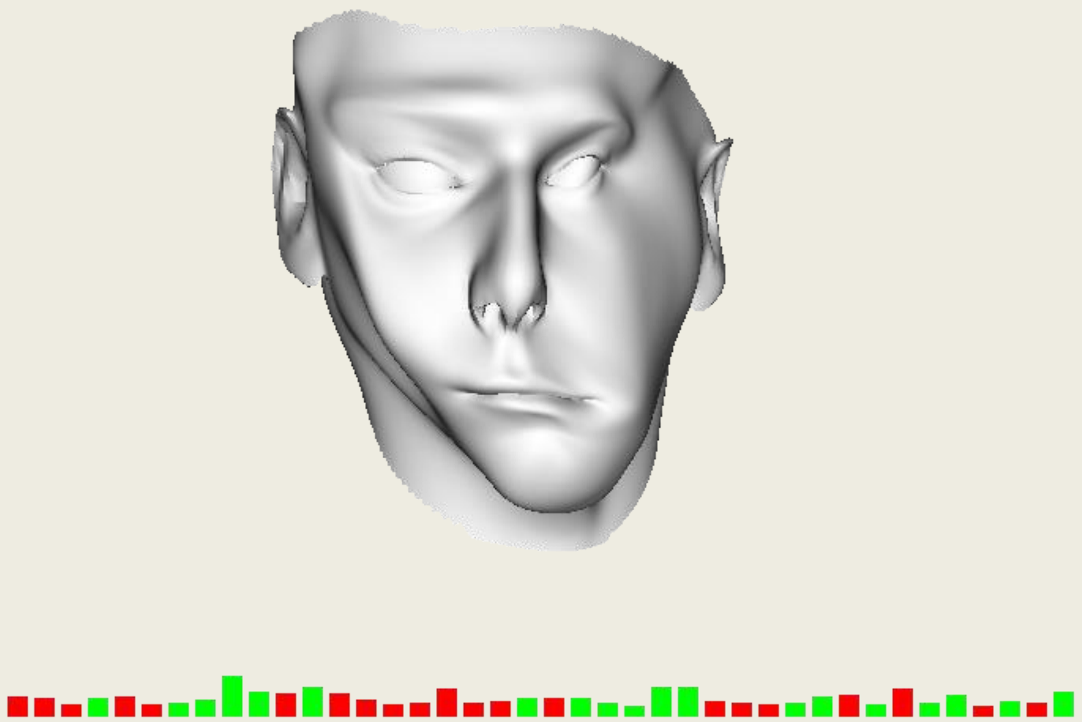
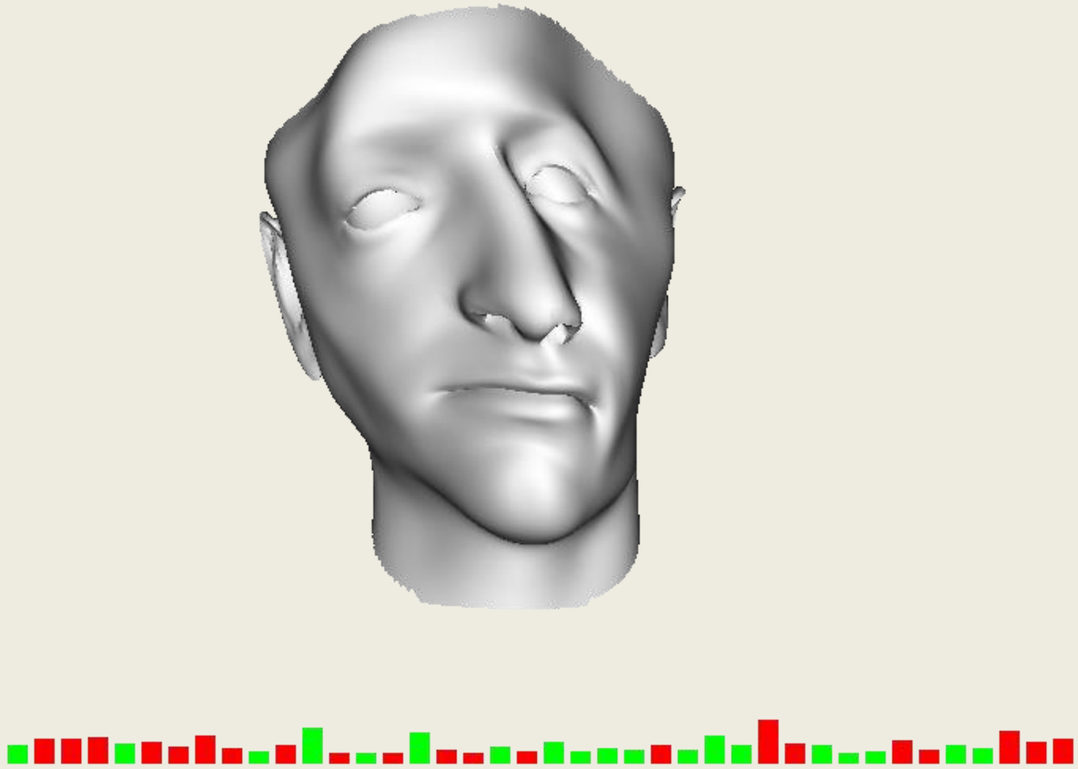
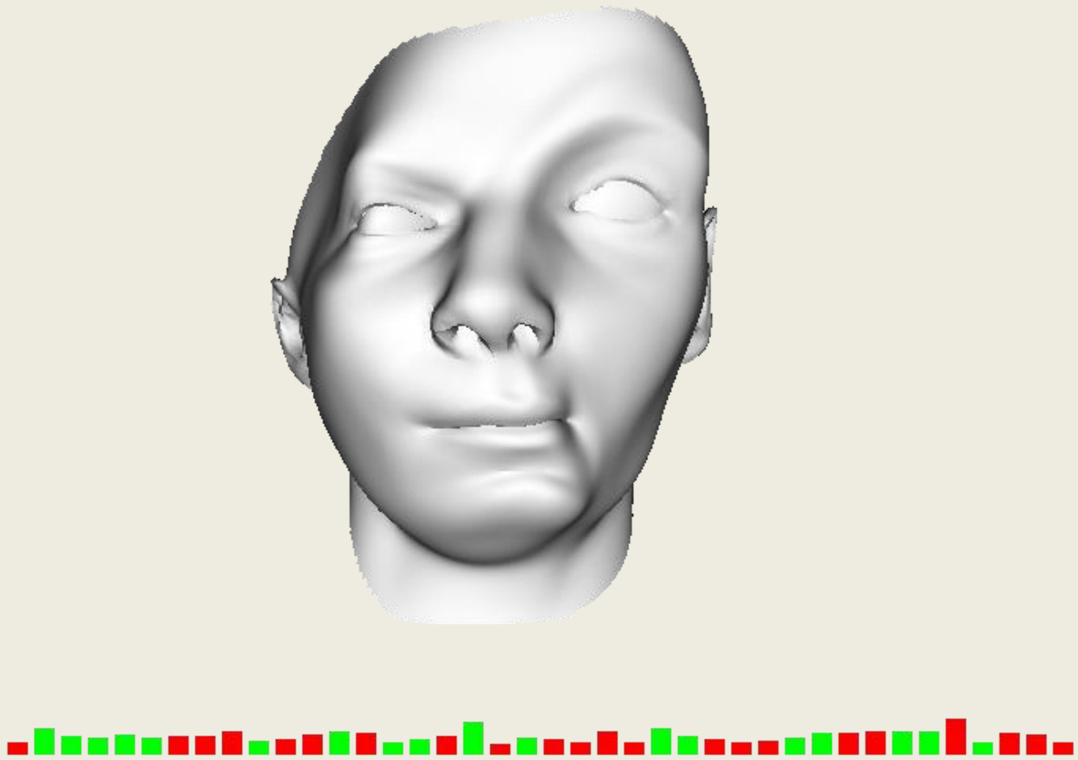




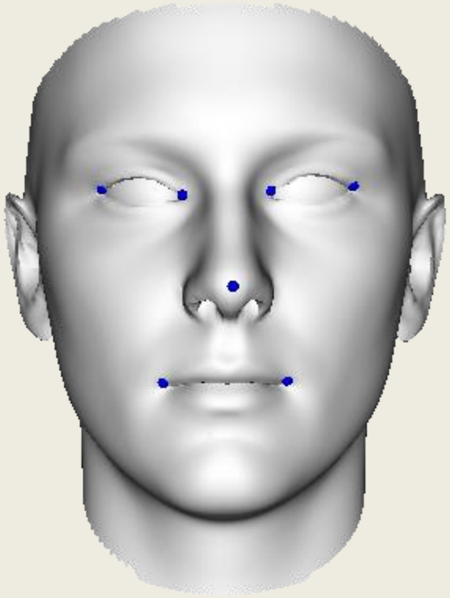


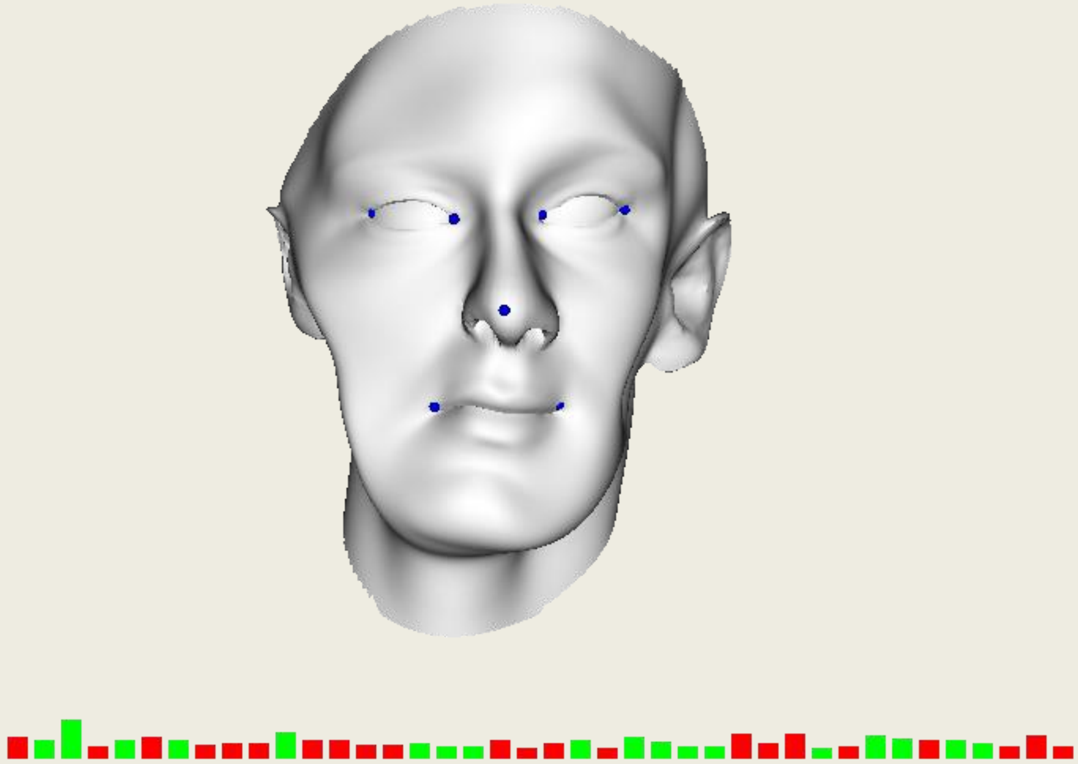


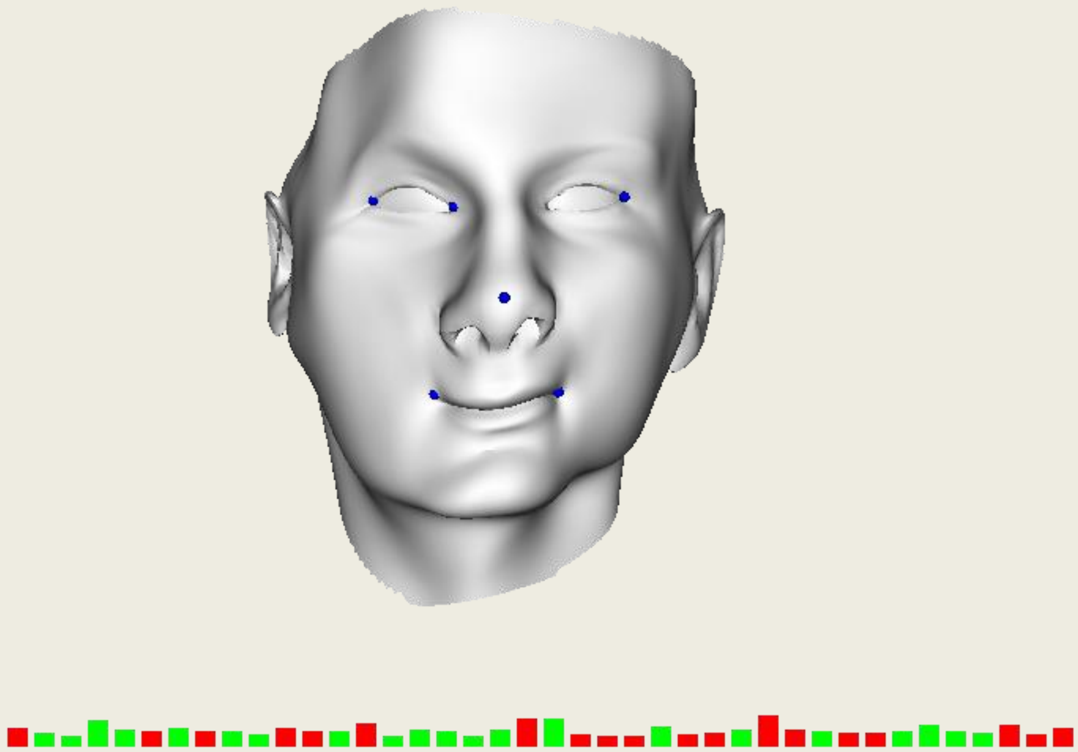


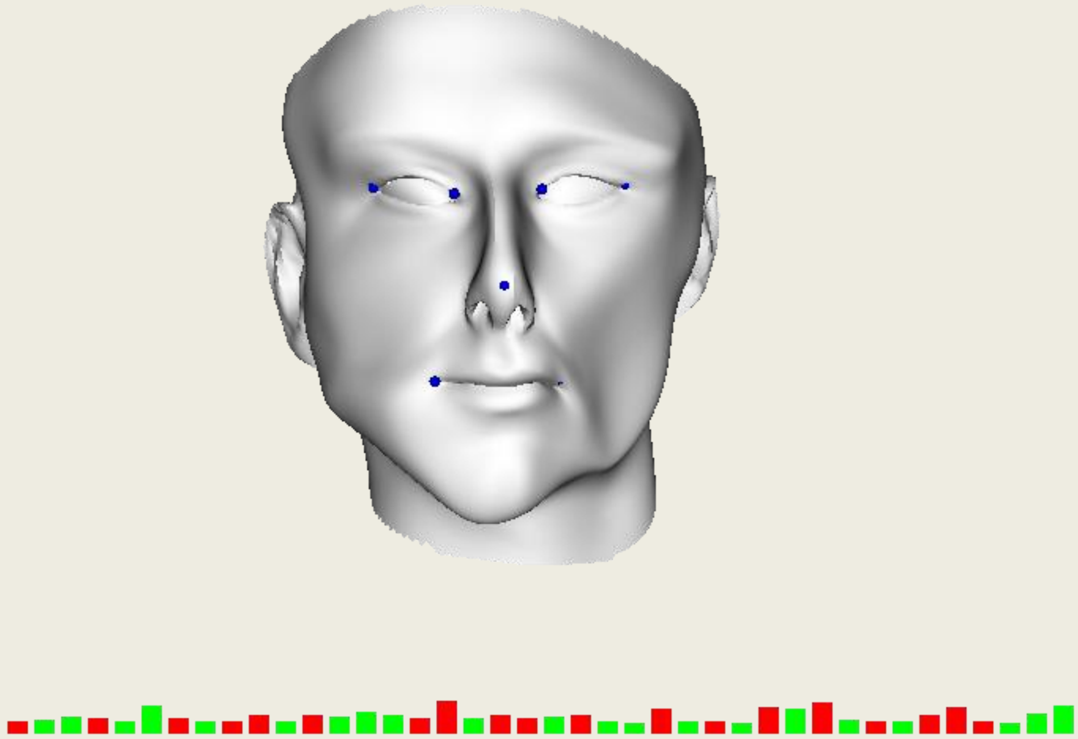


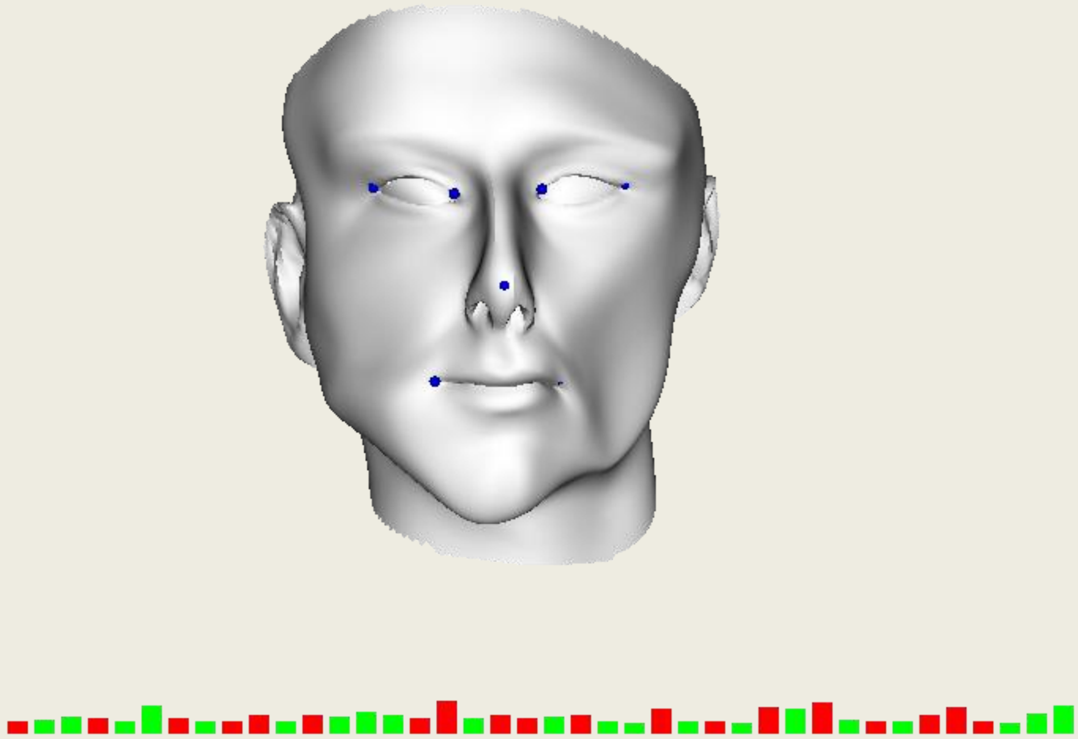
Advantage 2: Posterior models

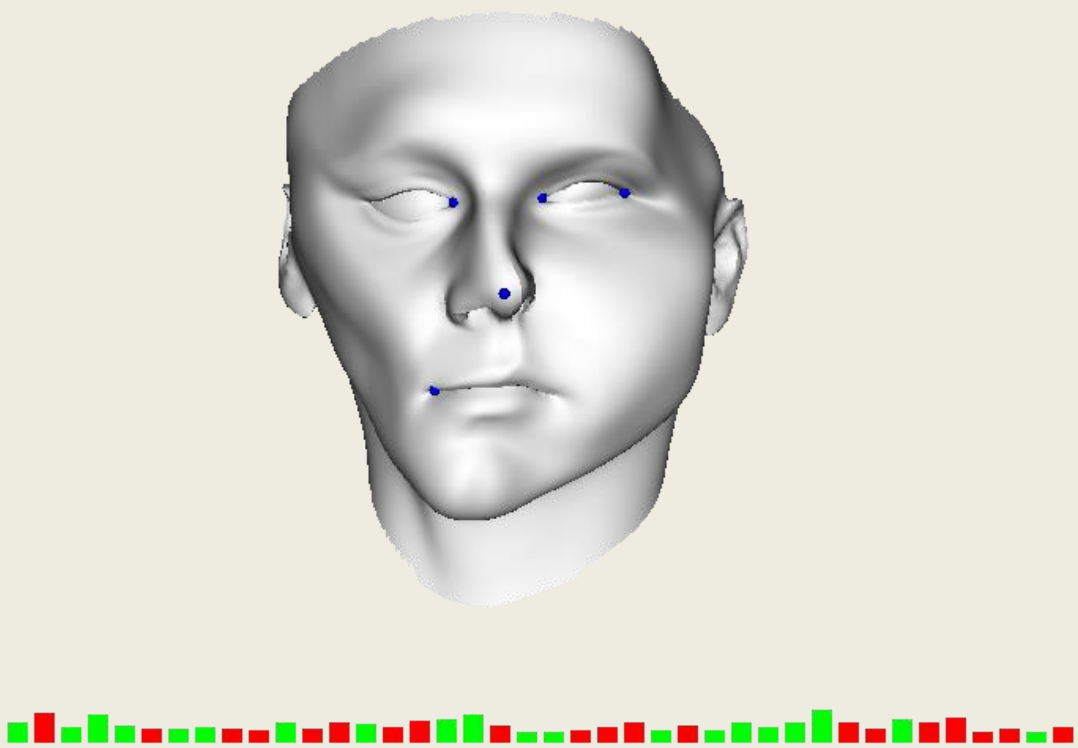












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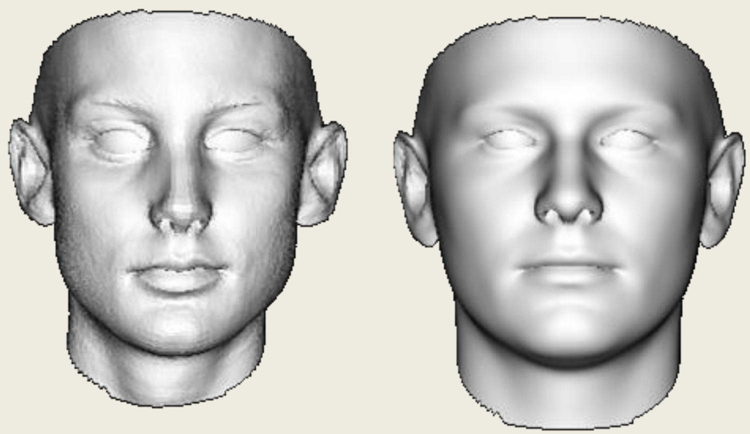
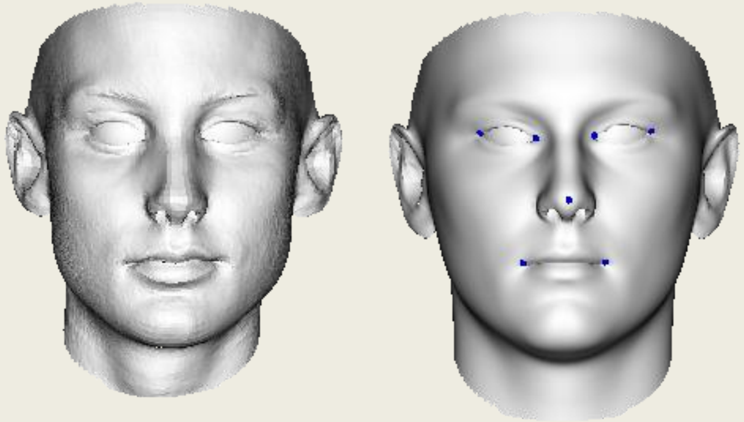
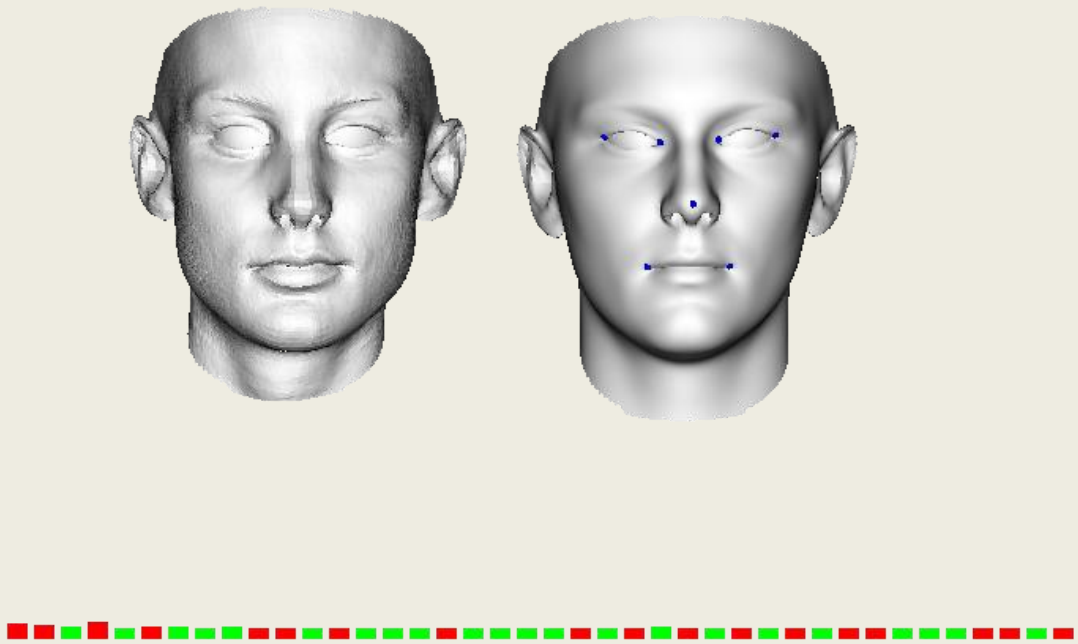
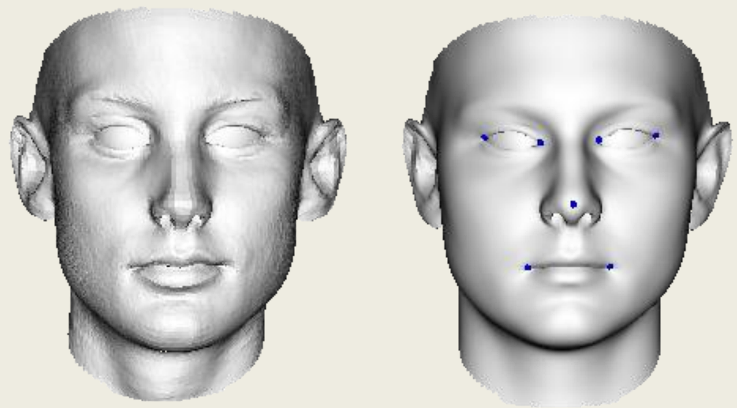
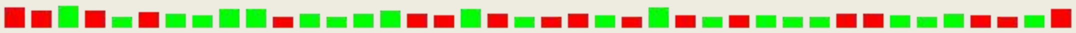
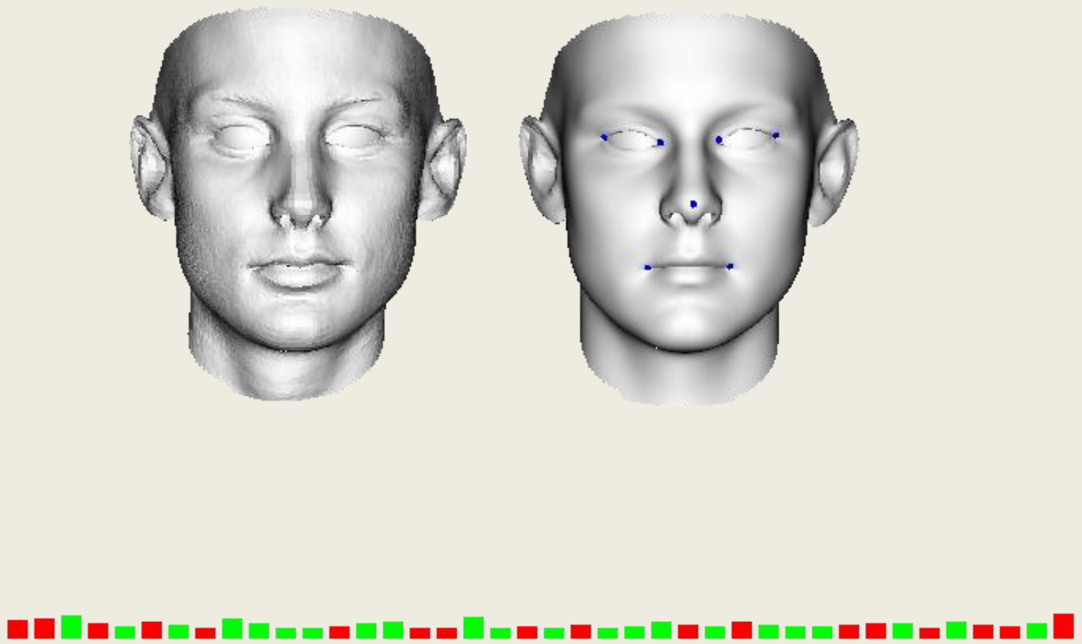


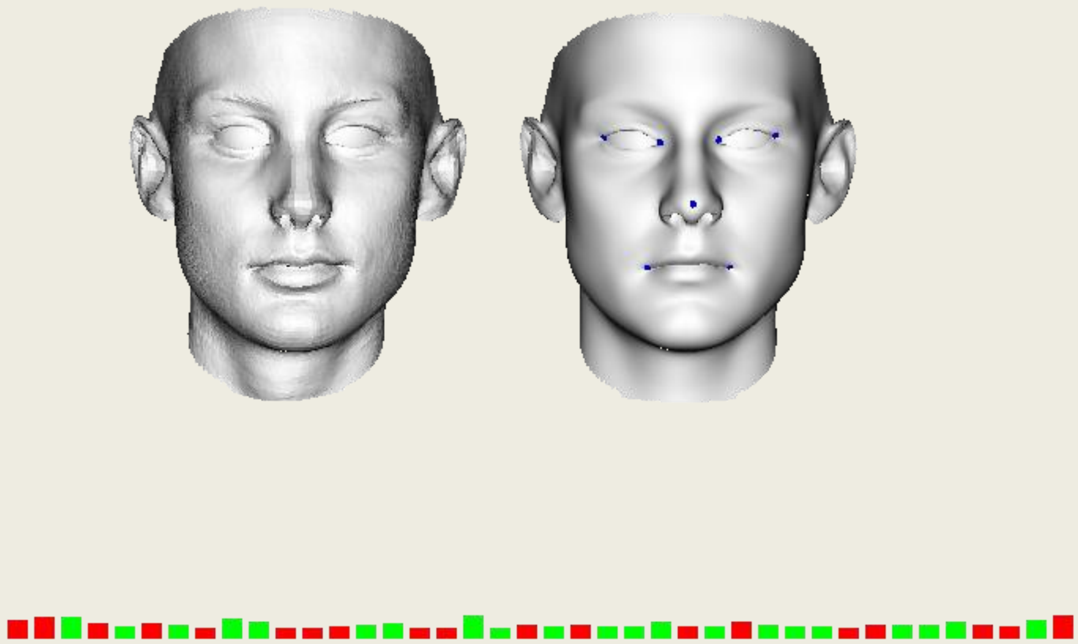


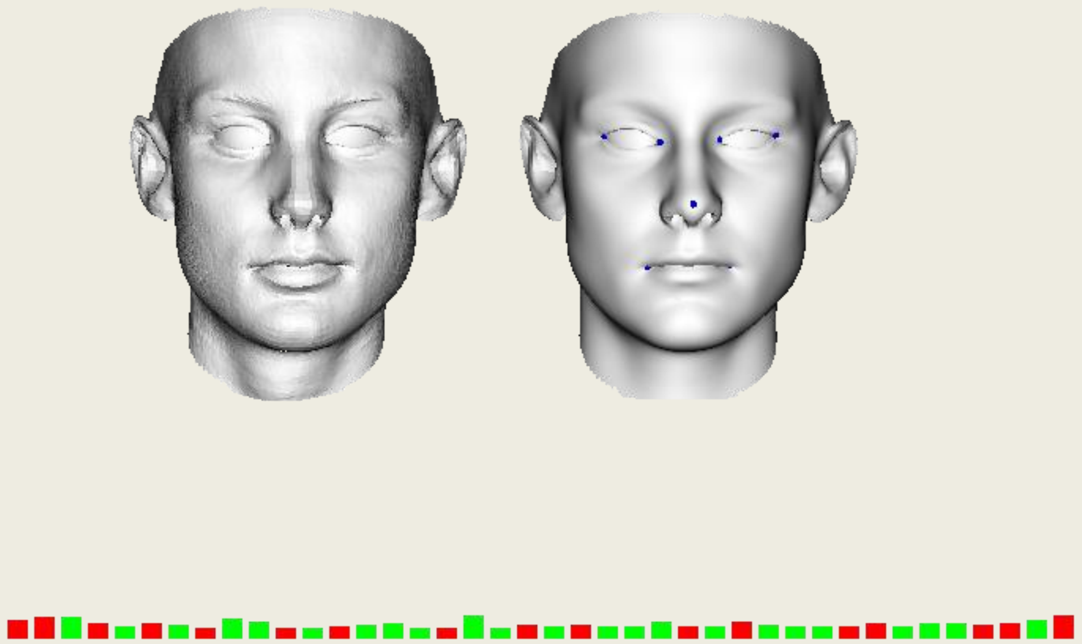








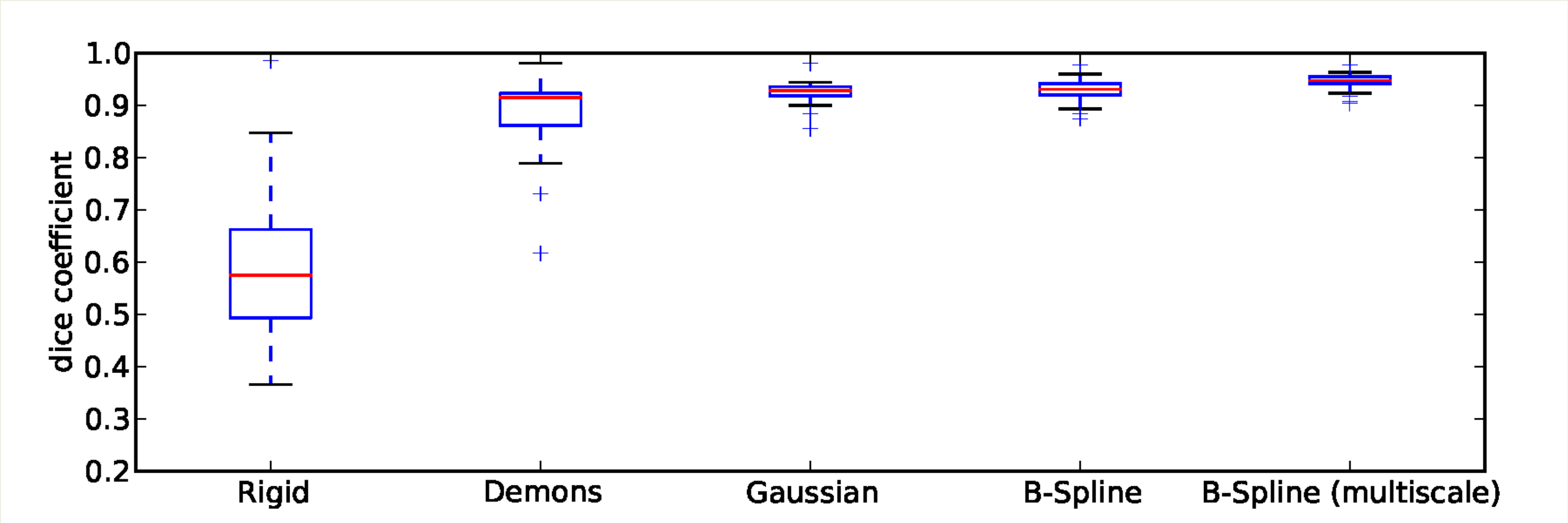




3D Image registration

Experimental Setup:

- 48 femur CT images
- Perform atlas matching
- Evaluation: dice coefficient with groundtruth segmentation



Conclusion

- Replaced non-rigid registration with model fitting
- One concept / one algorithm
  - Parametric, generative model
  - Works for images an surfaces
- Extreme flexibility in choice of prior
  - Any kernel can be used
  - Future work: Design application specific kernels

Thank you

Source code available at:

www.statismo.org