Curvature guided surface registration using level sets

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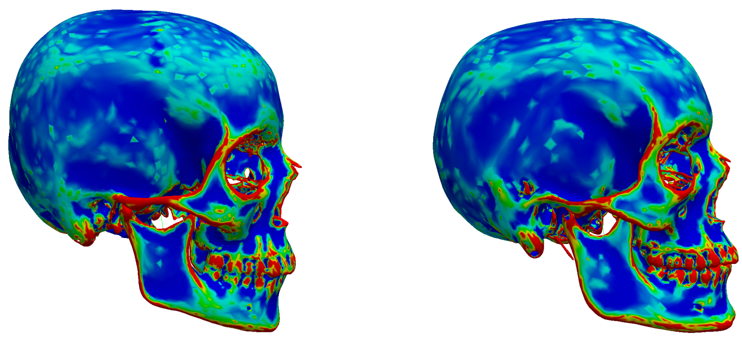
Abstract. We present a new approach for non-rigid surface-registration of 3D objects. By representing the surfaces as the zero-level of a signed distance function in 3D space, our algorithm becomes independent of topology. We propose an extension of Thirion's Demons algorithm to register the distance images. Our extension includes the use of a curvature term to guide the registration in the direction tangential to the zero level set. Our experiments show that our algorithm is suitable to register objects of complex topology, such as human skulls. Further, the curvature term leads to better correspondence in comparison to the standard Demons algorithm.

1 Introduction

Many questions in medical data analysis require the knowledge of the normal state of an anatomical structure. With statistical methods, applied to a set of exemplar data sets, it is possible to analyze and quantify the normal variability of such structures. In order to be able to extract meaningful information from different sample data, correspondence between the parts of the structure has to be established. The problem of establishing correspondence is known as the registration problem.

In this paper we consider the registration problem between two objects given as 2D surfaces in 3D space. In contrast to most other approaches for surface registration, we do not attempt to register the surfaces directly. Rather, we represent the surfaces as the zero level set of a signed distance function in 3D space. Correspondence is then established for the distance function, and thus in particular for the zero level set that represents the surface. This representation has the advantage that surfaces of arbitrary topology can be registered. Further, it yields meaningful correspondence in a neighborhood around the surface. For the registration we propose an extension of the well known Thirion's Demon algorithm [6]. We add a novel term that guides the registration on the surface such that points having similar curvature are matched. To illustrate the idea, figure 1 shows two skulls coloured according to their mean curvature. We observe that corresponding regions do indeed have similiar curvature.

While we believe that our method is applicable to a wide range of problems in medical imaging and computer vision, our particular motivation stems from two projects in medical data analysis. Namely, we aim at building statistical models of the human skull.



(a) (b)

Fig. 1: Two skulls colored according to their mean curvature.

2 Related work

A large amount of different registration methods can be found in the literature. For an overview of surface registration algorithms, we refer the reader to the comprehensive survey by Audette et al [1]. A detailed treatment of image registration methods can be found in the recent books of Yoo [8] and Modersitzki [3]. A precise mathematical interpretation of Thirion's Demons algorithm [6] can be found in [5]. Level set methods have been widely used for image segmentation and shape representation [4], and recently for image registration [7] [2].

3 Methods

We represent a surface $\Gamma \subset \mathbb{R}^3$ as the zero level set of an auxiliary function $I : \Omega \subset \mathbb{R}^3 \to \mathbb{R}$. The zero level set of I corresponds to the surface, i.e. $\Gamma = \{x \in \Omega \,|\, I(x) = 0\}$. A particularly convenient choice is to let I be the signed distance function to the surface. These functions can be interpreted as images and used as the input for an image registration algorithm.

As the basis for our registration we use Thirion's Demons algorithm, one of the most widespread algorithms for non-rigid image registration. Although originally formulated as an optical flow like algorithm, the Demons algorithm corrsponds essentially to the variational problem of minimizing the following functional (see [2] for more details)

$$\mathcal{J}[u] = \mathcal{D}[u] + \mathcal{R}[u]$$

where

$$\mathcal{D}[u] = \frac{1}{2} \int_{\Omega} \tfrac{1}{Q(x)} \left(I_0(x + u(x)) - I_1(x) \right)^2 dx$$

is a distance measure, and

$$\mathcal{R}[u] = \frac{1}{2} \sum_{l=1}^{3} \int_{\Omega} |\nabla u_l|^2 \, dx$$

a regularization term. Here I_0 and I_1 are the images and $u : \Omega \rightarrow \mathbb{R}^3$ a displacement field to be calculated. The weight Q is chosen as $Q(x) = |\nabla I_0(x)|^2 + (I_0(x) - I_1(x))^2$, motivated by Thirion's original formulation. Our experiments showed that when using the Demons Algorithm on the distance images, the images and in particular the level sets representing the surfaces are accurately matched. However, due to the lack of information on the surface, the correspondences implied by this matching are not always those a human expert would identify. Therefore, for our applications, we extend the Demons algorithm to incorporate curvature information by adding an additional curvature penalty term $\mathcal{C}$ to the cost functional. The functional becomes

$$\tilde{\mathcal{J}}[u] = \mathcal{D}[u] + \mathcal{R}[u] + \mathcal{C}[u]$$

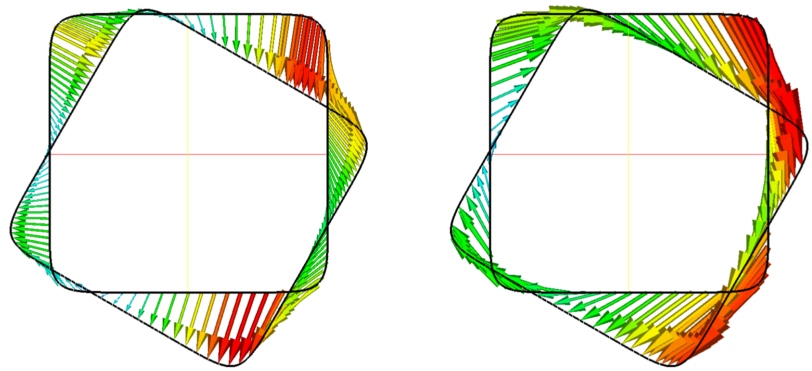
where

$$\mathcal{C}[u] = \frac{1}{2} \int_{\Omega} \tfrac{1}{Q_H(x)} \left(H_0(x + u(x)) - H_1(x)^2 \right) dx.$$

The term $\mathcal{C}$ is in fact the same as the distance measure $\mathcal{D}$, with the function I_i replaced by H_i. The function value $H_i(x) = \operatorname{div} \frac{\nabla I_i(x)}{|\nabla I_i(x)|}$ is the mean curvature of the level set of I_i at a point $x \in \Omega$. Thus, a minimizer of $\tilde{\mathcal{J}}$ does not only minimize the L^2-distance between the two images, but strives to match points with the same curvature. The new term also admits a natural interpretation in the original formulation of Thirion's Demons. It corresponds to the tangential component of the velocity term, which is zero in the original algorithm.
As is common in most image-based registration methods, we use a multiresolution approach to improve the robustness and convergence speed of the algorithm. Further, we calculate the curvature image from a smoothed version of the distance image in order to avoid an exaggerated influence of small features and noise.

4 Results

We performed experiments on different synthetic examples and medical data sets of the human femur-bone as well as the skull. Figure 2 illustrates the difference between the standard Demons algorithm and our extension. In both cases a dense correspondence between the two shapes is established. This is synonymous to a perfect matching of the shapes. However, the standard Demons algorithm matches only points that are close to the direction of the image-gradient. As a result the corners of one shape are not matched to those of the other shape but rather to the edges. Through the new curvature term, the curvature of the shapes, which is highest in the corners, is also considered and thus the corners of the shapes are matched correctly.
In Figure 3 we illustrate a possible application of the registration algorithm. We manually labelled the mandible in a reference skull, and registered the reference skull onto an unlabeled target skull. By applying the label of the nearest point in the reference skull, an automatic identification of the mandible in the target skull could be achieved.



(a) (b)

Fig. 2: Correspondence found by Thirion's Demons algorithm (a) and extended Demons algorithm (b). The original algorithm does not match corners to corners, while the extended one does.

5 Conclusion

We presented a new approach to surface registration, by representing a surface as a level set of a signed distance function and applying Thirion's Demons algorithm to the distance images. We found that this method accuratly matches the two surfaces, but that with the standard formulation of Thirion's Demons, the correspondences within the surfaces may not be optimal. By adding an additional curvature term, these problems could be alleviated. The new algorithm clearly yields better correspondences, while preserving the accuracy of the surface match. Our experiments confirmed that this method is suitable for our project and provides a clear advantage over the standard Demons algorithm. For registration of structures that are characterized by other features, the same technique can be applied to guide the registration process on the surface. Furthermore, the same approach can be used for any other image registration method which admits a variational formulation.

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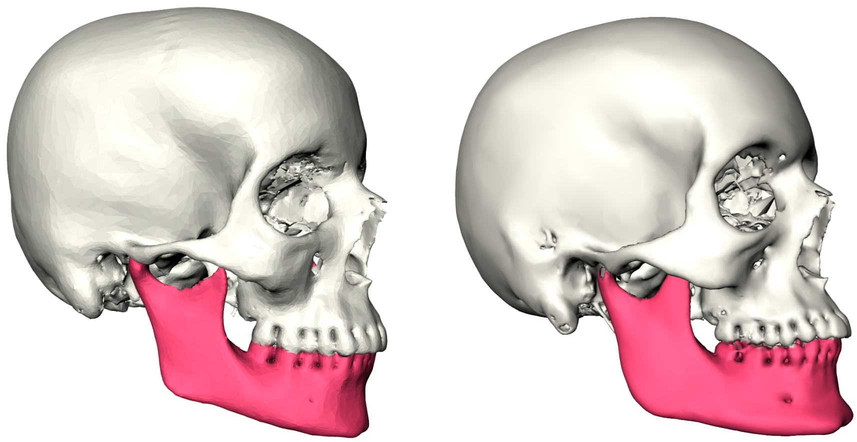

Fig. 3: The labelling of the mandible in the reference skull is transferred to a target skull.

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